LECTURE NOTES

On

ELECTRICAL MACHINE (Chapter 3)

Name of the Department- Electrical Engineering

SUBJECT CODE- TH 1

COURSE CONTENT

DC MOTOR

Explain Principle of working of a DC motor.

Explain concept of development of torque & back EMF in DC motor including simple problems.

Derive equation relating to back EMF, Current, Speed and Torque equation

Classify DC motors & explain characteristics, application.

State & explain three point & four point stator/static of DC motor by solid State converter.

Explain Speed of DC motor by field control and armature control method.

Explain power stages of DC motor & derive Efficiency of a DC motor.

MODULE III- DC MOTOR

Principle of Operation

DC motor operates on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force given by F = BIL newton. Where 'B' = flux density in wb, 'I' is the current and 'L' is the length of the conductor. The direction of force can be found by Fleming's left hand rule. From the point of construction, there is no difference between a DC generator and DC motor. Figure 3.1 shows a multipolar DC motor. Armature conductors are carrying current downwards under North Pole and upwards under South Pole. When the field coils are excited, with current carrying armature conductors, a force is experienced by each armature conductor whose direction can be found by Fleming's left hand rule. This is shown by arrows on top of the conductors. The collective force produces a driving torque which sets the armature into rotation. The function of a commutator in DC motor is to provide a continuous and unidirectional torque.

In DC generator the work done in overcoming the magnetic drag is converted into electrical energy. Conversion of energy from electrical form to mechanical form by a DC motor takes place by the work done in overcoming the opposition which is called the 'back emf'.

Back EMF and its Significance:

What is back emf:

When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence emf is induced in them as in a generator.

The induced emf acts in opposite direction to the applied voltage V (Lenz's law) and is known as ***Back EMF or Counter EMF (Eb).***

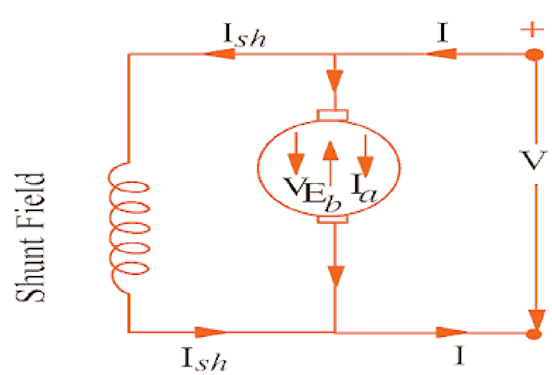
The equation for back emf in a DC motor is given below,

$$E_b = \frac{P\phi ZN}{60A}$$

The **back emf** Eb(= PΦZN/60 A) is always less than the applied voltage V, although this difference is small when the motor is running under normal conditions.

How Back EMF Occur in DC Motor:

Consider a shunt wound DC motor



Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back emf Eb is induced which opposes the applied voltage V. The applied voltage V has to force current through the armature against the back emf Eb. The electric work done in overcoming and causing the current to flow against Eb is converted into mechanical energy developed in the armature. It follows, therefore, that energy conversion in a dc motor is only possible due to the production of back emf Eb.

Net voltage across armature circuit = V – Eb

If Ra is the armature circuit resistance, then, Ia = (V – Eb)/Ra

Since V and Ra are usually fixed, the value of Eb will determine the current drawn by the motor.

If the speed of the motor is high, then back e.m.f. Eb (= PφZN/60 A) is large and hence the motor will draw less armature current and vice-versa.

The significance of Back EMF:

The presence of back emf makes the d.c. motor a *self-regulating machine* i.e., it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load.
Armature current (Ia),

$$I_a = \frac{V - E_b}{R_a}$$

When the motor is running on no load, small torque is required to overcome the friction and windage losses. Therefore, the armature current Ia is small and the back emf is nearly equal to the applied voltage.
If the motor is suddenly loaded, the first effect is to cause the armature to slow down. Therefore, the speed at which the armature conductors move through the field is reduced and hence the back emf Eb falls.
The decreased back emf allows a larger current to flow through the armature and larger current means increased driving torque.
Thus, the driving torque increases as the motor slows down. The motor will stop slowing down when the armature current is just sufficient to produce the increased torque required by the load.
If the load on the motor is decreased, the driving torque is momentarily in excess of the requirement so that armature is accelerated.
As the armature speed increases, the back emf Eb also increases and causes the armature current Ia to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load.
Therefore, the back emf in a DC motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

Torque Equation

When armature conductors of a DC motor carry current in the presence of stator field flux, a mechanical torque is developed between the armature and the stator. Torque is given by the product of the force and the radius at which this force acts.

Torque T = F × r (N-m) …where, F = force and r = radius of the armature
Work done by this force in once revolution = Force × distance = F × 2πr
(where, 2πr = circumference of the armature)
Net power developed in the armature = word done / time
= (force × circumference × no. of revolutions) / time
= (F × 2πr × N) / 60 (Joules per second)
But, F × r = T and 2πN/60 = angular velocity ω in radians per second.
Putting these in the above equation
Net power developed in the armature = P = T × ω (Joules per second) = $\frac{2\pi N}{60}$ ×T = NT/9.55

Armature torque (Ta)

The power developed in the armature can be given as

Pa = Ta × ω = Ta × 2πN/60 rpm

The mechanical power developed in the armature is converted from the electrical power,
Therefore, mechanical power = electrical power
That means, Ta × 2πN/60 = Eb.Ia

Or $T_a = \frac{E_b I_a}{\frac{2\pi N}{60}} = 9.55\frac{E_b I_a}{N}$ N-m

The above equation shows the relationship between armature torque, back emf, flux and speed of a DC motor.

We know, Eb = PΦNZ / 60A

Therefore, Ta × 2πN/60 = (PΦNZ / 60A) × Ia

Rearranging the above equation,
Ta = (PZ / 2πA) × Φ.Ia (N-m) = $0.159\frac{P\phi Z I_a}{A}$ N-m

The term (PZ / 2πA) is practically constant for a DC machine. Thus, armature torque is directly proportional to the product of the flux and the armature current i.e. Ta ∝ Φ.Ia
a. In case of DC series motor ϕ is directly promotional to Ia, therefore Ta ∝ Ia^2

b. For shunt motor, ϕ is practically constant, hence $T_a \propto I_a$

Shaft Torque (Tsh)

Due to iron and friction losses in a dc machine, the total developed armature torque is not available at the shaft of the machine. Some torque is lost, and therefore, shaft torque is always less than the armature torque.

Shaft torque of a DC motor is given as,

T_{sh} = output in watts / ($2\pi N/60$) ....(where, N is speed in RPM)

$= 9.55\frac{Output}{N}$ N-m

The difference $T_a - T_{sh}$ is known as lost torque.

Speed of a DC Motor

We Know $E_b = V - I_aR_a$ or $\frac{P\emptyset ZN}{60A} = V - I_aR_a$

$\therefore N = \frac{V - IaRa}{\emptyset} \times \frac{60A}{ZP}$ rpm

$= \frac{Eb}{\emptyset}\frac{60A}{ZP}$ rpm or $N = k\frac{Eb}{\emptyset}$ or $N \propto \frac{Eb}{\emptyset}$

For series motor:

Let N_1, N_2 = speed in the 1st and 2nd case

I_{a1}, I_{a2} = armature current in the 1st and 2nd case

Φ_1, ϕ_2 = flux/pole in the 1st and 2nd case

$N_1 \propto \frac{E_{b1}}{\emptyset 1}$ where $E_{b1} = V - I_{a1}R_a$ and $N_2 \propto \frac{E_{b2}}{\emptyset 2}$ where $E_{b2} = V - I_{a2}R_a$

$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\emptyset 1}{\emptyset 2}$ as in series motor $\emptyset \propto I_a$ $\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$

For shunt motor: $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\emptyset 1}{\emptyset 2}$

Classification of DC Motor

The **types of DC motor** include:

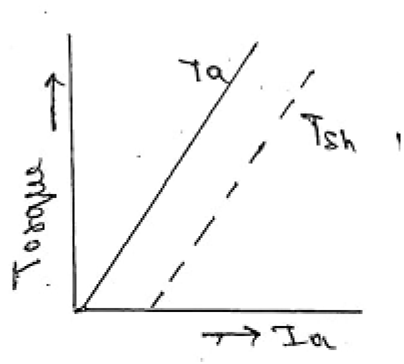
1. Separately Excited DC Motor
2. Self Excited DC Motor
   a. Shunt Wound DC Motor
   b. Series Wound DC Motor
   c. Compound Wound DC Motor
      i. Short shunt DC Motor
      ii. Long shunt DC Motor

Characteristics of DC shunt motor

Armature torque vs armature current T_a vs I_a characteristics

For a shunt motor flux can be assumed practically constant (through at heavy loads, ϕ decreases somewhat due to increased armature reaction), hence $T_a \propto I_a$

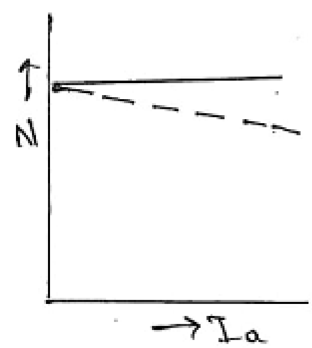
Therefore electrical characteristic is shown below, is practically a straight line through the origin. Shaft torque is shown as dotted line.



Torque Current Characteristic of DC shunt motor

Speed vs armature current N_a vs I_a characteristics

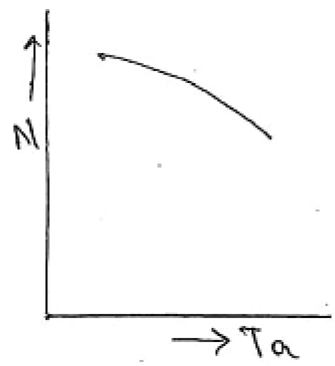
As flux ϕ is assumed to be constant, we can say $N \propto Eb$. But, as back emf is also almost constant, the speed should remain constant. But practically, ϕ as well as Eb decreases with increase in load. Back emf Eb decreases slightly more than ϕ, therefore, the speed decreases slightly. Generally, the speed decreases only by 5 to 15% of full load speed. Therefore, a shunt motor can be assumed as a constant speed motor. In speed vs. armature current characteristic in the following figure, the straight horizontal line represents the ideal characteristic and the actual characteristic is shown by the dotted line.



Speed vs armature current characteristics of DC shunt motor

Speed vs armature torque N_a vs T_a characteristics

From T_a vs I_a and N_a vs I_a with increase with torque the speed of DC shunt motor decreases. The nature of the characteristics is drooping in nature shown in figure as given below.



Speed vs armature torque characteristics of DC shunt motor

Characteristics of DC series motor

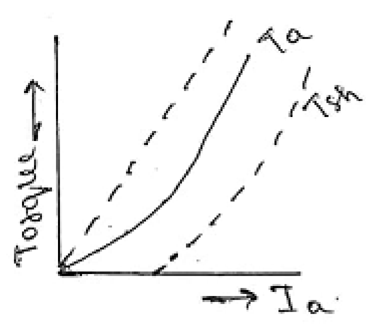
Armature torque vs armature current characteristics

We know that $T_a \propto \phi I_a$. In case of DC series motor as field windings also carry the armature current

$\phi \propto I_a$

$\therefore T \propto I^2$

At light loads, I_a and hence ϕ is small. But as I_a increases T_a increases as the square of the current up-to saturation. After saturation ϕ becomes constant, the characteristic becomes a straight line as shown in Figure below. Therefore a series motor develops a torque proportional to the square of the armature current. This characteristic is suited where huge starting torque is required for accelerating heavy masses.



Torque Current Characteristic of DC series motor

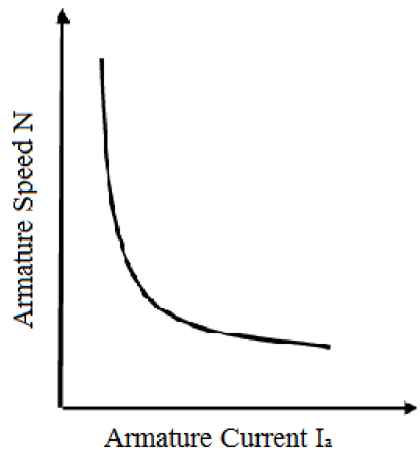
Speed vs armature current characteristics

$N \propto E_b/\phi$

In DC series motor $I_a \propto \phi$

Therefore $N \propto 1/ I_a$

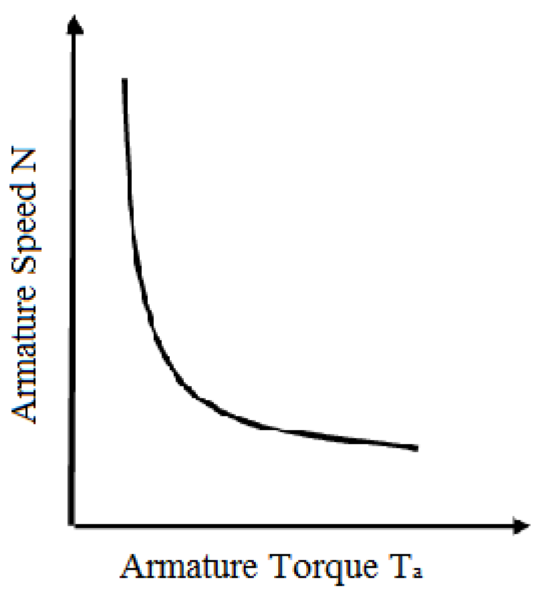
If I_a increases, speed decreases. This characteristic is shown in figure below. Therefore the speed is inversely proportional to armature current I_a. When load is heavy I_a ia heavy thus speed is low. When load is low I_a is low thus speed becomes dangerously high. Hence series motor should never start without load on it.



Speed vs armature current characteristics of DC series motor

Speed vs armature torque characteristics

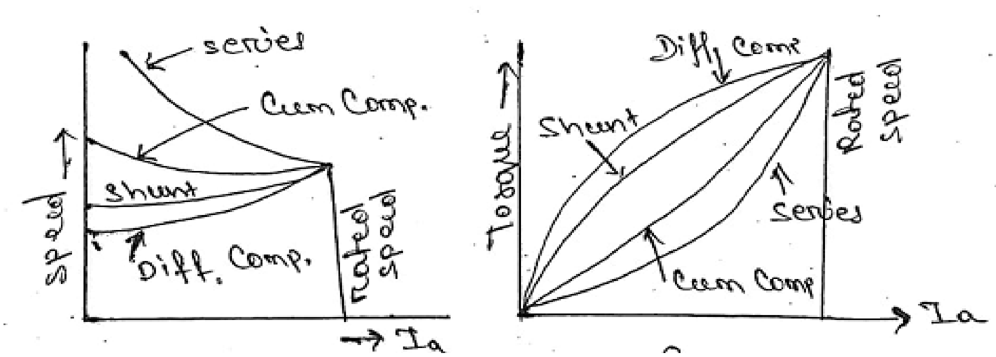
From T_a vs I_a and N_a vs I_a characteristics Speed is inversely proportional to torque. The characteristic is shown in figure as given below.



Speed vs armature torque characteristics of DC series motor

Characteristics of DC compound motor

There are two different types of compound motors in common use, they are the cumulative compound motor and the differential compound motor. In the cumulative compound motor, the field produced by the series winding aids the field produced by the shunt winding. The speed of this motor falls more rapidly with increasing current than does that of the shunt motor because the field increases. In the differential compound motor, the flux from the series winding opposes the flux from the shunt winding. The field flux, therefore, decreases with increasing load current. Because the flux decreases, the speed may increases with increasing load. Depending on the ratio of the series-to-shunt field ampere-turns, the motor speed may increases very rapidly.



Characteristics of DC compound motors

Application of DC motors

Application of DC shunt motor

The characteristics of a DC shunt motor give it a very good speed regulation, and it is classified as a constant speed motor, even though the speed does slightly decrease as load is increased. Shunt wound motors are used in industrial and automotive applications where precise control of speed and torque are required.

Application of DC series motor

For a given input current, the starting torque developed by a DC series motor is greater than that developed by a shunt motor. Hence series motors are used where huge starting torques are necessary. Ex. Cranes, hoists, electric traction etc. The DC series motor responds by decreasing its speed for the increased in load. The current drawn by the DC series motor for the given increase in load is lesser than DC shunt motor. The drop in speed with increased load is much more prominent in series motor than that in a shunt motor. Hence series motor is not suitable for applications requiring a constant speed.

Application of DC compound motor

Cumulative compound wound motors are virtually suitable for almost all applications like business machines, machine tools, agitators and mixers etc. Compound motors are used to drive loads such as shears, presses and reciprocating machines.

Differential compound motors are seldom used in practice (because of rising speed characteristics).

Starting of DC Motor

Necessity of starter:

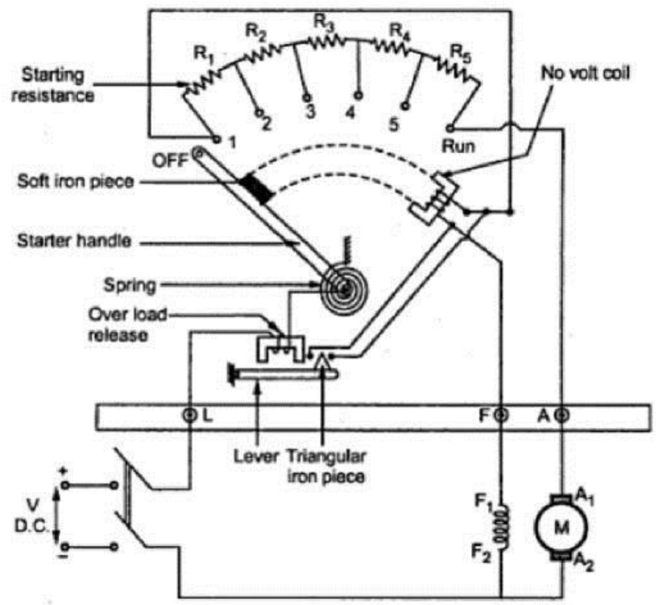
The current drawn by the armature is given by $I_a = \frac{V_t - E_b}{R_a}$

At starting, as N=0 so Eb = 0 thus $I_a = \frac{V_t}{R_a}$

Armature resistance will be very low. Therefore, the current drawn by the motor will be very high. In order to limit this high current, a starting resistance is connected in series with the armature. The starting resistance will be excluded from the circuit after the motor attains its rated speed. From there on back emf limits the current drawn by the motor.

Three Point Starter

The arrangement is shown in the figure below shows a three point starter for shunt motor.



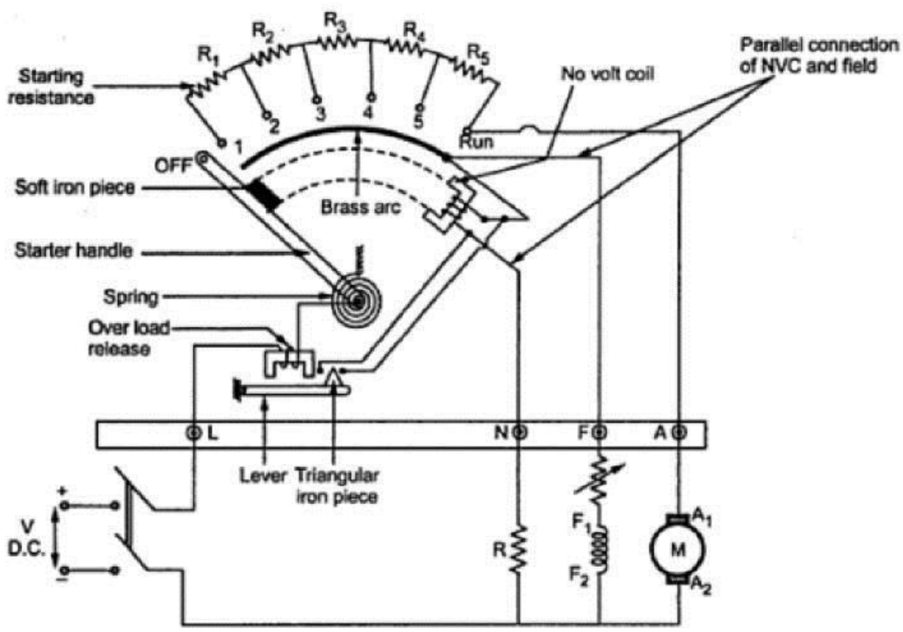
Internal view of three point starter

It consists of resistances arranged in steps, R_1 to R_5 connected in series with the armature of the shunt motor. Field winding is connected across the supply through a protective device called 'NO – Volt Coil'. Another protection given to the motor in this starter is 'over load release coil'. To start the motor the starter handle is moved from OFF position to Run position gradually against the tension of a hinged spring. An iron piece is attached to the starter handle which is kept hold by the No-volt coil at Run position. The function of No volt coil is to get de-energized and release the handle when there is failure or disconnection or a break in the field circuit so that on restoration of supply, armature of the motor will not be connected across the lines without starter resistance. If the motor is over loaded beyond a certain predetermined value, then the electromagnet of overload release will exert a force enough to attract the lever which short circuits the electromagnet of No volt coil. Short circuiting of No volt coil results in de-energisation of it and hence the starter handle will be released and return to its off position due to the tension of the spring.

If it is desire to control the speed of the motor in addition, then a field rheostat is connected in the field circuit. The motor speed can be increased by weakening the flux ($N \propto 1/\phi$). But there is one difficulty for control speed with this arrangement. If too much resistance is cut in by the field rheostat, then field current is reduced too much so that it is enable to create enough electromagnetic pull to overcome the spring tension. Hence the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-sped motor. This can be overcome with four-point starter.

Four Point Starter

One important change is the No Volt Coil has been taken out of the shunt field and has been connected directly across the line through a Protecting resistance 'R'. When the arm touches stud one. The current divides into three paths, 1. Through the starter resistance and the armature, 2. Through shunt field and the field rheostat and 3.Through No-volt Coil and the protecting resistance 'R'. With this arrangement, any change of current in shunt field circuit does not affect the current passing though the NO-volt coil because, the two circuits are independent of each other. Thus the starter handle will not be released to its off position due to changes in the field current which may happen when the field resistance is varied. Fig given below shows internal view of 4-point starter.



Internal view of four point starter

Losses and efficiency of DC Machines

Efficiency of a DC motor: $\eta = \frac{Output}{Input} = \frac{Input - Losses}{Input}$

Various losses occurring in a DC machine are listed below- Total losses can be broadly divided into two types.

1) Constant losses

2) Variable losses

These losses can be further divided as

1) Constant losses –

i) Core loss or iron loss

a) Hysteresis loss

b) Eddy current loss

ii) Mechanical loss

a) Windage loss

b) Friction loss – brush friction loss and Bearing friction loss.

2) Variable losses –

i) copper loss ($I^2 r$)

a) Armature copper loss

b) Field copper loss

c) Brush contact loss

ii) Stray load loss

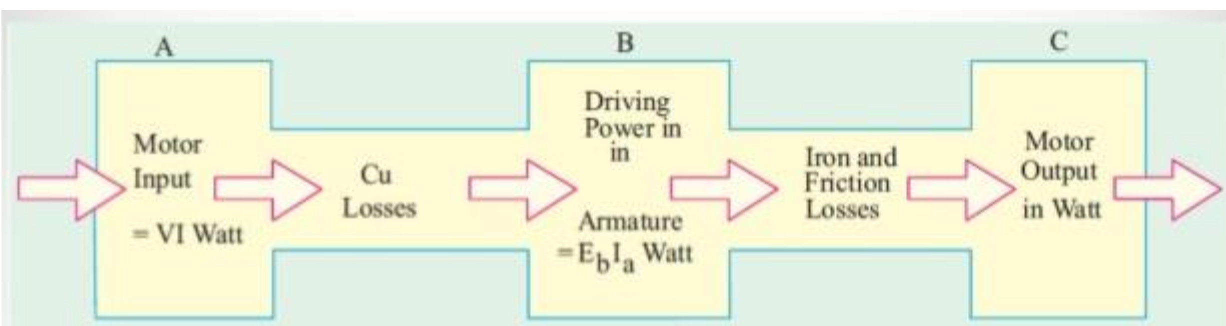
a) Copper stray load loss

b) Core stray load loss

Core loss or iron loss occurs in the armature core is due to the rotation of armature core in the magnetic flux produced by the field system. Iron loss consists of a) Hysteresis loss and b) Eddy current loss.

Power stages of DC Motor:

Power flow in a DC generator is shown in figure



Power flow in a DC motor

Overall or commercial Efficiency= C/A

Electrical Efficiency = B/A

Mechanical Efficiency = C/B

A-B = Cu Losses and B – C = iron and friction losses

CONDITION FOR MAXIMUM EFFICIENCY

Generator output = VI_L --------------where V is the terminal voltage and I_L is load current.

Generator input = VI_L + losses = $VI_a + I_a^2 r + P_c$

Where $I_a^2 r$ = Cu loss also called as variable loss; P_c= Iron loss also called as constant loss

If the shunt field current is negligible, then $I_a = I_L$

Hence Generator input = $V I_L + I_L^2 r + P_c$

Generator Efficiency: $\eta = \frac{Output}{Input} = \frac{VI_L}{VI_L + I_L^2 R_a + P_C}$

Efficiency will be maximum when $\frac{d}{dI_L}\eta = 0$ OR $I_L^2 R_a = P_C$

Hence efficiency is maximum when variable loss is equal to constant loss.

The load current corresponding to maximum efficiency is $I_L = \sqrt{\frac{P_c}{r_a}}$

Speed Control of DC Motor

Speed of a DC motor can be controlled in a wide range.

$$N = \frac{E_b}{K\emptyset} = \frac{V - I_a R_a}{K\emptyset}$$

The speed equation shows that speed can be controlled by-

1.Variation of field current which varies the flux/pole and is known as field control.

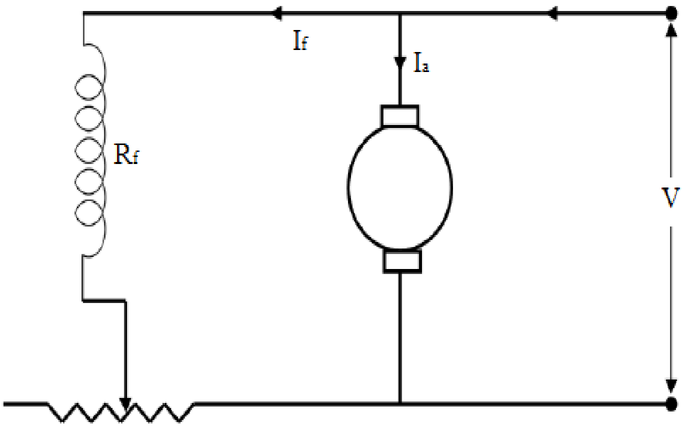
2. Variation of armature resistance known as armature voltage control.

3. Variation of terminal voltage 'V' known as Ward Leonard method.

Speed control of Shunt motor:

1. *Flux control method:* It is already explained above that the **speed of a dc motor** is inversely proportional to the flux per pole. Thus by decreasing the flux, speed can be increased and vice versa.

To control the flux, a rheostat is added in series with the field winding, as shown in the circuit diagram. Adding more resistance in series with the field winding will increase the speed as it decreases the flux. In shunt motors, as field current is relatively very small, I_{sh}^2R loss is small. Therefore, this method is quite efficient. Though speed can be increased above the rated value by reducing flux with this method, it puts a limit to maximum speed as weakening of field flux beyond a limit will adversely affect the commutation.



Circuit diagram for speed control using field control method

2. Armature control method: **Speed of a dc motor** is directly proportional to the back emf E_b and $E_b = V - I_aR_a$. That means, when supply voltage V and the armature resistance R_a are kept constant, then the speed is directly proportional to armature current I_a. Thus, if we add resistance in series with the armature, I_a decreases and, hence, the speed also decreases. Greater the resistance in series with the armature, greater the decrease in speed

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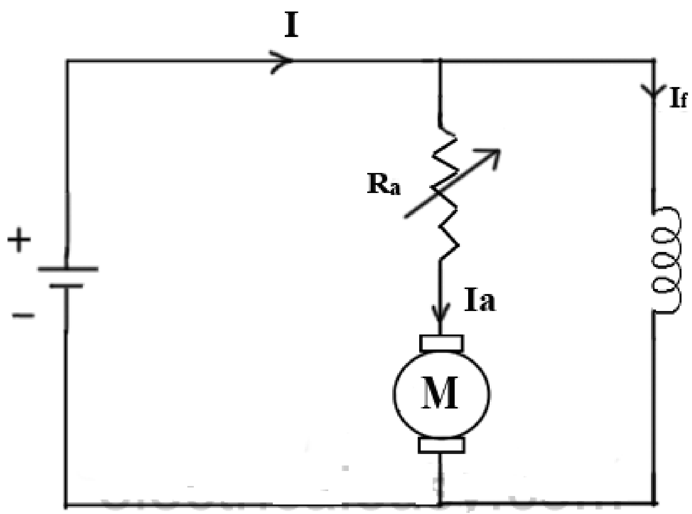


Circuit diagram for speed control using armature control method

Speed control of series motor

1. Flux control method:

a. Field diverters: variable resistance is connected parallel to the series field as shown in fig (a). This variable resistor is called as a diverter, as the desired amount of current can be diverted through this resistor and, hence, current through field coil can be decreased. Thus, flux can be decreased to the desired amount and speed can be increased.

b. Armature diverter: Diverter is connected across the armature as shown in fig (b).
For a given constant load torque, if armature current is reduced then the flux must increase, as $T_a \propto Ø I_a$
This will result in an increase in current taken from the supply and hence flux Ø will increase and subsequently speed of the motor will decrease.

c. Tapped field control: As shown in fig (c) field coil is tapped dividing number of turns. Thus we can select different value of Ø by selecting different number of turns.

d. Paralleling field coils: In this method, several speeds can be obtained by regrouping coils as shown in fig (d).

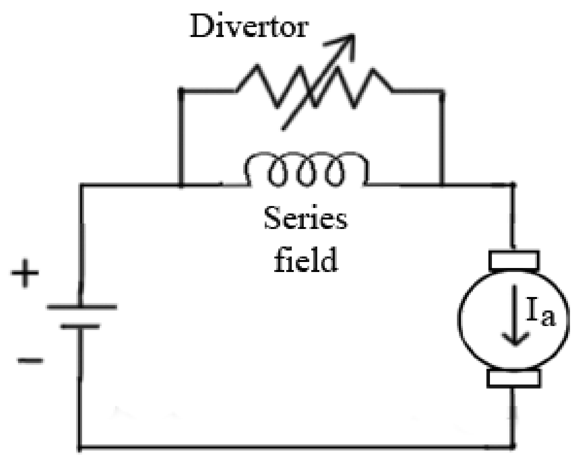


fig (a) Field Divertor

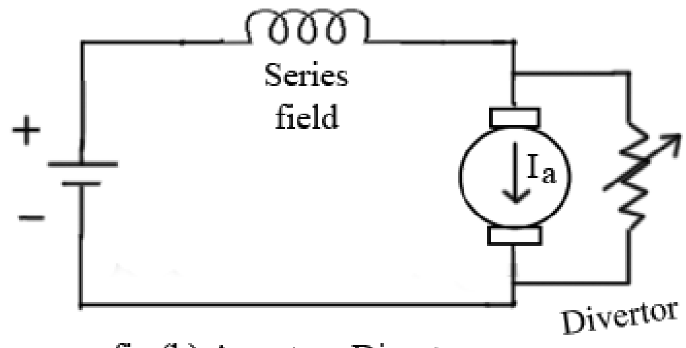


fig (b) Armature Divertor

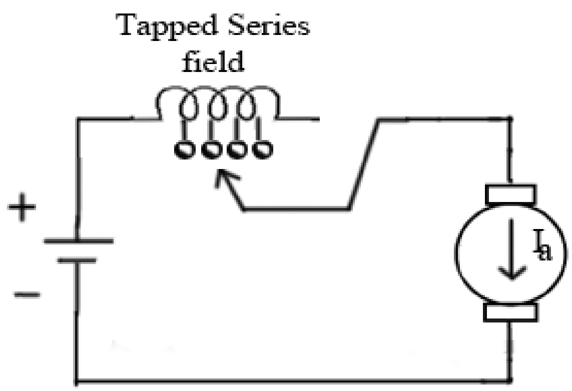


fig (c) Tapped field

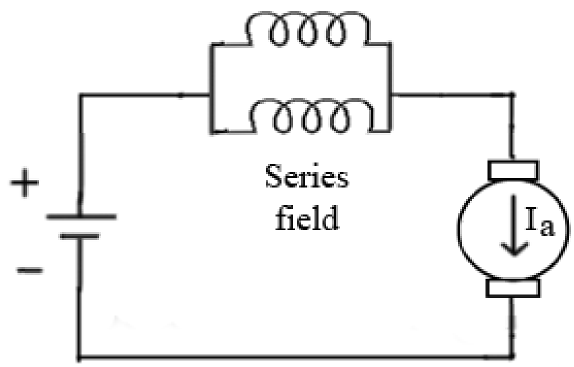
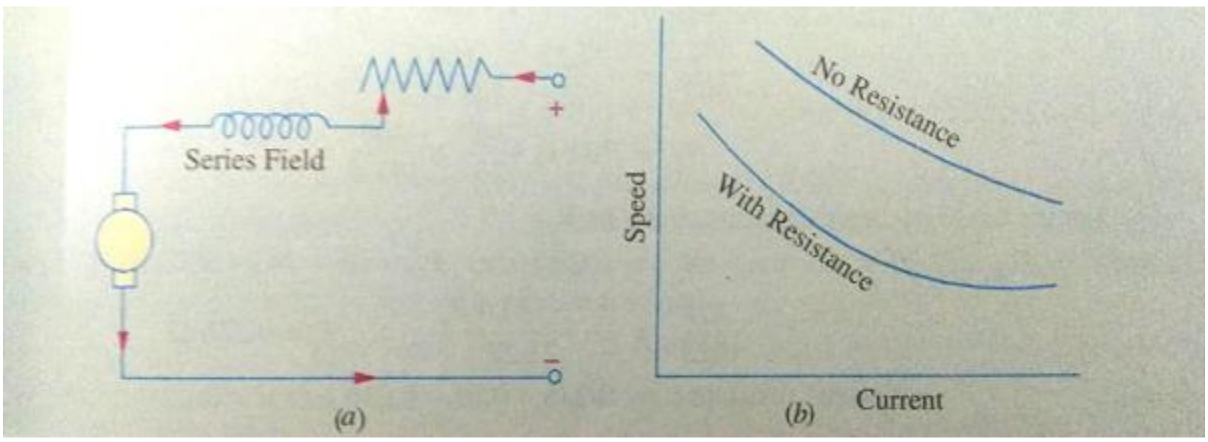


fig (d) Paralleling Field coils

2. ***Variable Resistance in series with Motor:***

By increasing the resistance in series with the armature as shown in figure given below the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.



Example 1: A 4-pole wave wound dc series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N-m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.

Solution:

$$T_a = 0.159\,\phi Z I_a\,(P/A)\ \text{N-m}$$
$$209 = 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a\,(4/2);\ I_a = 20.1\ \text{A}$$
$$E_a = V - I_a R_a = 500 - 20.1 \times 3 = 439.7\ \text{V}$$

$$E_b = \Phi Z N \times (P/A) \text{ or } 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times 2$$
$$N = 6.73\ \text{r.p.s. or } 382.2\ \text{r.p.m.}$$

Example 2:

A 230V shunt motor delivers 30hp at the shaft at 1120rpm. If the motor has an efficiency of 87% at this load, determine:

a) The total input power.
b) The line current.

Solution

(a) $\eta = \frac{P_{o/p}}{P_{i/p}}$

$$P_{i/p} = \frac{P_{o/p}}{\eta}$$

$$= \frac{30 * 746}{0.87}$$ (output power= 30 Hp= 30×746 =22,380 W)

$$= 25.72\ Kw$$

$$P_{i/p} = 25.72\ Kw$$

(b) $P_{i/p} = V_t I_l$

Hence $I_l = 25720/230 = 111.82$ A

Example2. A 250 volt DC shunt motor has armature resistance of 0.25 ohm on load it takes an armature current of 50A and runs at 750rpm. If the flux of the motor is reduced by 10% without changing the load torque, find the new speed of the motor.

Solution:

Given data V = 250

$R_a = 0.25$

$I_a = 50$

$N_1 = 750$ rpm

$\Phi_2 = 90\%\Phi_1$

For shunt motor $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} = \frac{\Phi_1}{\Phi_2}$$

$E_{b1} = V - I_{a1}R_a = 250-(50x0.25) = 237.5V$

$E_{b2} = V - I_{a2}R_a$

Load torque is constant

$Ta_1 = Ta_2$

Or $\Phi_1 Ia_1 = \Phi_2 Ia_2$

Or $\Phi_1 \times 50 = 0.9\ \Phi_1\ Ia_2$ hence $I_{a2} = 55.55A$

$E_{b2} = 250-55.55X0.25 = 236.12V$

N_2 = 828 rpm

Example 3. *A 230-V d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 115 Ω. At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load.*

Example 3.

Solution. $N_1 = 1200$ r.p.m., $E_{b1} = 230 - (0.5 \times 2.5) = 228.75$ V

$N_2 = 1120$ r.p.m., $E_{b2} = 230 - 0.5\ I_{a2}$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \therefore \quad \frac{1120}{1200} = \frac{230 - 0.5 I_{a2}}{228.75}; \quad I_{a2} = \mathbf{33\ A}$$

Line current drawn by motor $= I_{a2} + I_{sh} = 33 + (230/115) = \mathbf{35\ A}$

Power input at rated load $= 230 \times 35 = \mathbf{8{,}050\ W}$

Example 4. A dc motor takes an armature current of 110A at 480 V. The armature circuit resistance is 0.2 ohm. The machine has 6 poles and the armature is lap connected with 864 conductors. The flux per pole is 0.05 Wb . Calculate (i) the speed and (ii) gross torque developed by the armature?

Solution:

$$E_b = 480 - 110 \times 0.2 = 458 \text{ V}, \quad \Phi = 0.05 \text{ W}, Z = 864$$

$$E_b = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ or } 458 = \frac{0.05 \times 864 \times N}{60} \times \left(\frac{6}{6}\right)$$

$$N = 636 \text{ r.p.m.}$$

$$T_a = 0.159 \times 0.05 \times 864 \times 110\ (6/6) = \mathbf{756.3\ N\text{-}m}$$

Example 5:

A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.

Solution. As Generator [Fig. (a)]

Output current $= 25{,}000/250 = 100$ A ; $I_{sh} = 250/100 = 2.5$ A ; $I_a = 102.5$ A

Generated e.m.f. $= 250 + I_a R_a = 250 + 102.5 \times 0.06 = 256.15$ V

Power developed in armature $= E_b I_a = \dfrac{256.15 \times 102.5}{1000} = 26.25$ kW

As Motor [Fig (b)]

Motor input current $= 100$ A ; $I_{sh} = 2.5$ A, $I_a = 97.5$ A

$E_b = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15$ V

Power developed in armature $= E_b I_a = 244.15 \times 97.5/1000 = 23.8$ kW

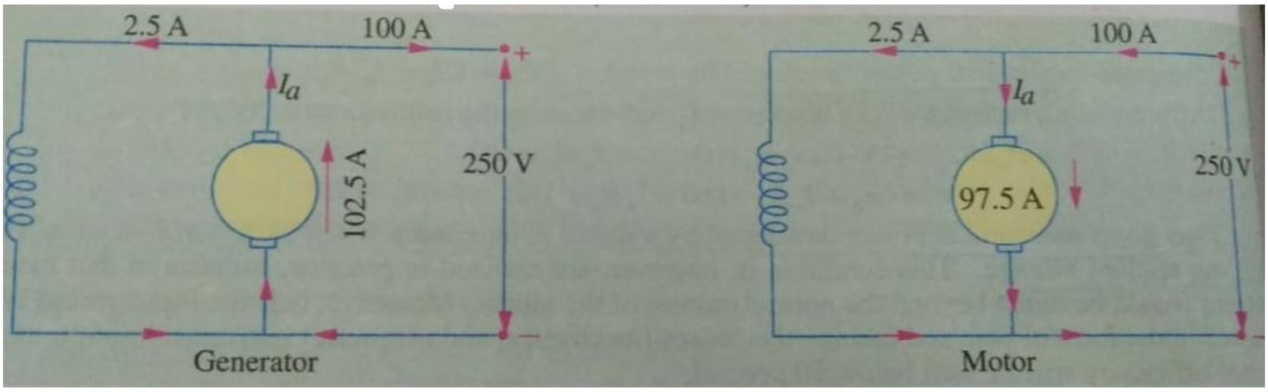


Figure (a) Figure (b)

Example 6:

Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

Solution. Developed torque or gross torque is the same thing as armature torque.

$\therefore \quad T_a = 0.159\, \Phi ZA\, (P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45\, (4/2) = 286.2$ N-m

$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193$ V

Now, $\quad E_b = \Phi ZN\, (P/A)$ or $193 = 25 \times 10^{-3} \times 800 \times N \pi \times (4/2)$

$\therefore \quad N = 4.825$ r.p.s.

Also, $\quad 2\pi N T_{sh} =$ output or $2\pi \times 4.825\, T_{sh} = 8200 \qquad \therefore T_{sh} = 270.5$ N-m

Example 7:

A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that $R_a = 0.2$ Ω.

Solution. As seen from Art 27.7, $T_a \propto \Phi I_a$. Since Φ is constant, $T_a \propto I_a$

$\therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2} \quad \therefore \quad T_{a2}/T_{a1} = I_{a2}/I_{a1}$

$\therefore \quad 2 = I_{a2}/50$ or $I_{a2} = 100$ A

Now, $N_2/N_1 = E_{b2}/E_{b1}$ — since Φ remains constant.

$E_{b1} = 220 - (50 \times 0.2) = 210$ V $\qquad E_{b2} = 220 - (100 \times 0.2) = 200$ V

$\therefore \quad N_2/500 = 200/210 \qquad \therefore N_2 =$ **476 r.p.m.**

Example 8:

A 4-pole, 240 V, wave connected shunt motor gives 1119 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole (d) rotational losses and (e) efficiency.

Solution.

$$E_b = V - I_a R_a - \text{brush drop} = 240 - (50 \times 0.1) - 2 = 233 \text{ V}$$

Also

$$I_a = 50 \text{ A}$$

(a) $$\text{Armature torque } T_a = 9.55 \frac{E_b I_a}{N} \text{ N-m} = 9.55 \times \frac{233 \times 50}{1000} = \mathbf{111 \text{ N-m}}$$

(b) $$T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{11{,}190}{1000} = \mathbf{106.9 \text{ N-m}}$$

(c) $$E_b = \frac{\Phi Z N}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

$$\therefore \quad 233 = \frac{\Phi \times 540 \times 1000}{60} \times \left(\frac{4}{2}\right) \quad \therefore \quad \mathbf{\Phi = 12.9 \text{ mWb}}$$

(d) Armature input $= V I_a = 240 \times 50 =$ **12,000 W**

Armature Cu loss $= I_a^2 R_a = 50^2 \times 0.1 = 250$ W ; Brush contact loss $= 50 \times 2 = 100$ W

$\therefore$ Power developed $= 12{,}000 - 350 = 11{,}650$ W ; Output $= 11.19$ kW $= 11{,}190$ W

$\therefore$ Rotational losses $= 11{,}650 - 11{,}190 =$ **460 W**

(e) Total motor input $= VI = 240 \times 51 = 12{,}340$ W ; Motor output $= 11{,}190$ W

$$\therefore \quad \text{Efficiency} = \frac{11{,}190}{12{,}240} \times 100 = \mathbf{91.4\ \%}$$