LECTURE NOTES

ON

ELECTRICAL MACHINE (Chapter 4)

BRANCH- ELECTRONICS AND TELLECOMMUNICATION

4th Semester

Department of Electrical Engineering

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ROURKELA 12

COURSE CONTENTS

CHAPTER-4

AC CIRCUITS

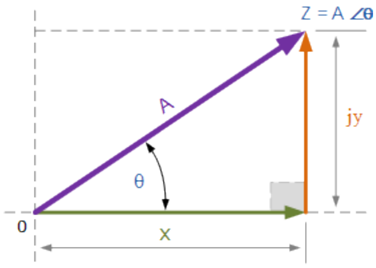
4.1 State Mathematical representation of phasors, significant of operator "J"
4.2 Discuss Addition, Subtraction, Multiplication and Division of phasor quantities.
4.3 Explain AC series circuits containing resistance, capacitances, Conception of active, reactive and apparent power and Q-factor of series circuits & solve related problems.
4.4 Find the relation of AC Parallel circuits containing Resistances, Inductance and Capacitances Q- factor of parallel circuits.

Phasor Algebra :

The 'Phasor' is defined as "The complex number in the polar form with which we can analyze the circuit". It is a vector quantity. In this vector representation we use Cartesian plane.

A vector quantity can be expressed in terms of

(i) Rectangular or Cartesian form
(ii) Trigonometric form
(iii) Exponential form
(iv) Polar form



(i) Rectangular or Cartesian form

In the rectangular form, the phasor is divided up into a real part, x and an imaginary part, y forming the generalised expression Z= x + jy

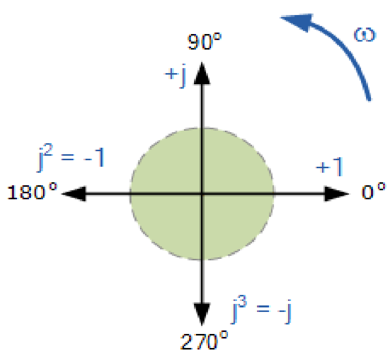
Where x = A cosθ is the active part and y = A sinθ is the reactive part
j is an operator which shift the phasor by an angle of 90° in counter-clock wise direction.

90° rotation: $j^1 = \sqrt{-1} = +j$

180° rotation: $j^2 = \left(\sqrt{-1}\right)^2 = -1$

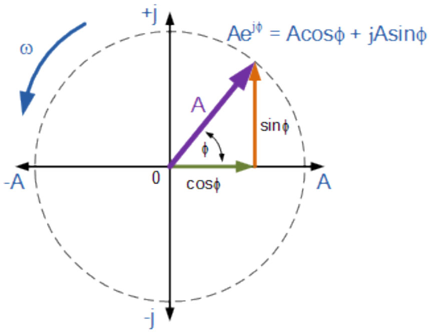
270° rotation: $j^3 = \left(\sqrt{-1}\right)^3 = -j$

360° rotation: $j^4 = \left(\sqrt{-1}\right)^4 = +1$



(ii)Trigonometric form

$Z = A(\cos\theta + j\sin\theta\ )$



(iii) Exponential form :-

Z= A $e^{j\theta}$ where $\theta = \tan^{-1}\frac{y}{x}$ and $A=\sqrt{(x^2+y^2)}$

(iv) Polar form :- **Z= A∠$\boldsymbol{\theta}$**

$$A^2 = x^2 + y^2$$

$$A = \sqrt{x^2 + y^2}$$

$$\text{Also,} \quad x = A.\cos\theta, \quad y = A.\sin\theta$$

Complex Addition and Subtraction

$$A = x + jy \qquad B = w + jz$$

$$A + B = (x + w) + j(y + z)$$

$$A - B = (x - w) + j(y - z)$$

Multiplication and Division of Complex Numbers

$$A \times B = (4 + j1)(2 + j3)$$

$$= 8 + j12 + j2 + j^2 3$$

$$\text{but } j^2 = -1,$$

$$= 8 + j14 - 3$$

$$A \times B = 5 + j14$$

Multiplication in Polar Form

$$Z_1 \times Z_2 = A_1 \times A_2 \angle\theta_1 + \theta_2$$

Multiplying together 6 ∠30° and 8 ∠– 45° in polar form gives us.

$$Z_1 \times Z_2 = 6 \times 8 \angle 30^o + (-45^o) = 48 \angle -15^o$$

Division in Polar Form

$$\frac{Z_1}{Z_2} = \left(\frac{A_1}{A_2}\right)\angle\theta_1 - \theta_2$$

$$\frac{Z_1}{Z_2} = \left(\frac{6}{8}\right)\angle 30^0 - (-45^0) = 0.75\angle 75^0$$

Example1: Find $|-1+4i|$.

Sol: $|-1+4i| = \sqrt{1+16} = \sqrt{17}$

Examples2. Write the following complex numbers in trigonometric form:

(a) $-4+4i$

To write the number in trigonometric form, we need A and θ.

$A = \sqrt{-4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$\tan\theta = \frac{4}{-4} = -1$

hence $\theta = 3\pi/4$ Then, $-4+4i = 4\sqrt{2}\,(\cos 3\pi/4 + j\sin 3\pi/4)$

(b) $2 - j\frac{2\sqrt{3}}{3}$

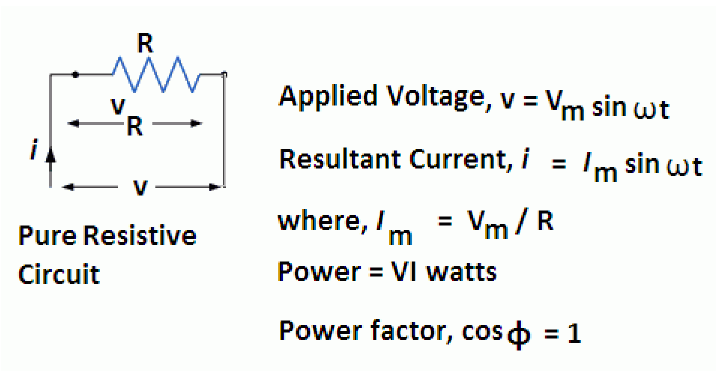
$$E = \sqrt{\left[2^2 + \left(\frac{2\sqrt{3}}{3}\right)\right]} = \frac{4\sqrt{3}}{3}$$

$\tan\theta = \frac{-\frac{2\sqrt{3}}{3}}{2} = -\sqrt{3}/3$ hence $\theta = 11\pi/6$,

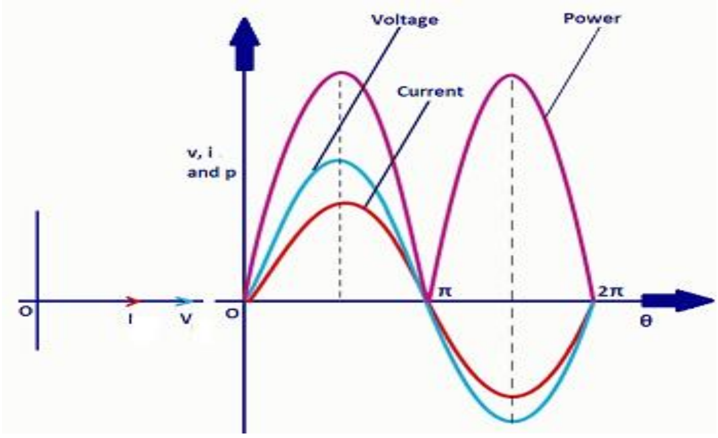
Then, the trigonometric form is $\frac{4\sqrt{3}}{3}(\cos 11\pi/6 + j\sin 11\pi/6)$

Purely Resistive Circuit

In a purely resistive circuit whole of the applied voltage is utilized in overcoming the ohmic resistance of the circuit. A *purely resistive circuit* is also known as the non-inductive circuit.



From the expression of instantaneous applied voltage and instantaneous current it is evident that in a **purely resistive circuit**, the applied voltage and current are in phase with each other.

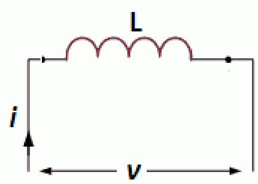


It is seen from the **power curve for purely resistive circuit** no part of power cycle becomes negative at any time i.e. in the purely resistive circuit power is never zero. This is so because instantaneous values of voltage and current are always either positive or negative and hence the product is always positive. The frequency of power cycle is double that of the voltage and current waves.

The power factor of the purely resistive circuit (cos φ) is 1.

Purely Inductive Circuit

A pure inductive coil is that which has no ohmic resistance and hence no I^2R loss.



Pure Inductive Circuit

Applied Voltage, $v = V_m \sin \omega t$

Resultent Current, $i = I_m \sin(\omega t - \frac{\pi}{2})$

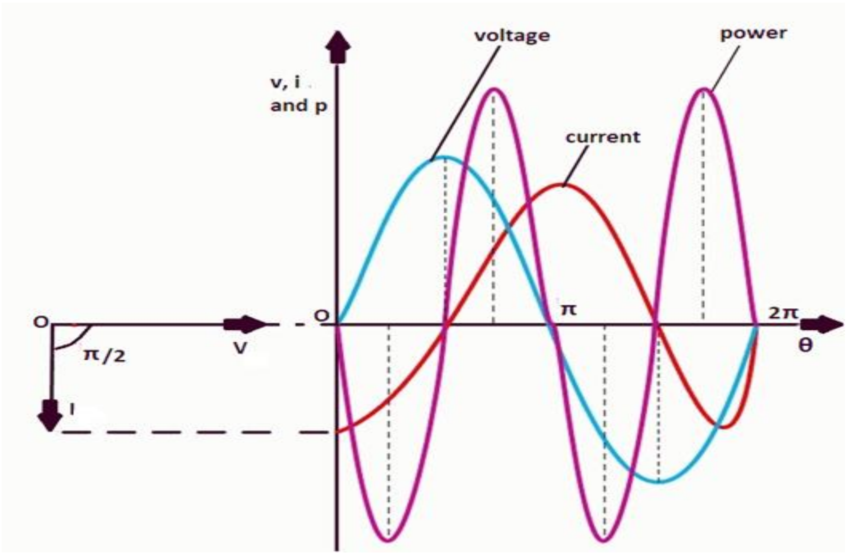
Where, $I_m = \frac{V_m}{X_L}$

Inductive Reactance, $X_L = 2\pi f L$ ohms

Power absorbed by circuit = 0

Power factor, $\cos \phi = 0$

From the expression of instantaneous applied voltage and instantaneous current flowing through the purely inductive circuit, it is observed that the current lags behind the voltage by π/2.

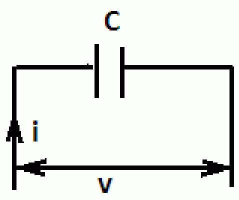


The power curve for the purely inductive circuit is shown in above figure. It is clear that average power in a half cycle is zero as the negative and positive loop area under power curve is the same.

In a purely inductive circuit, during the first quarter cycle, what so ever energy (or power) is supplied by the source that is stored in the magnetic field set-up around the coil. In the next quarter cycle, the magnetic field collapses and the energy (or power) stored in the magnetic field is returned to the source. Hence, no power is consumed in a purely inductive circuit.

<u>Purely Capacitive Circuit</u>

When an alternating voltage is applied to a purely capacitive circuit, the capacitor is charged first in one direction and then in the opposite direction.



Pure Capacitive Circuit

Applied Voltage, $v = V_m \sin \omega t$

Resultent Current, $i = I_m \sin(\omega t + \frac{\pi}{2})$

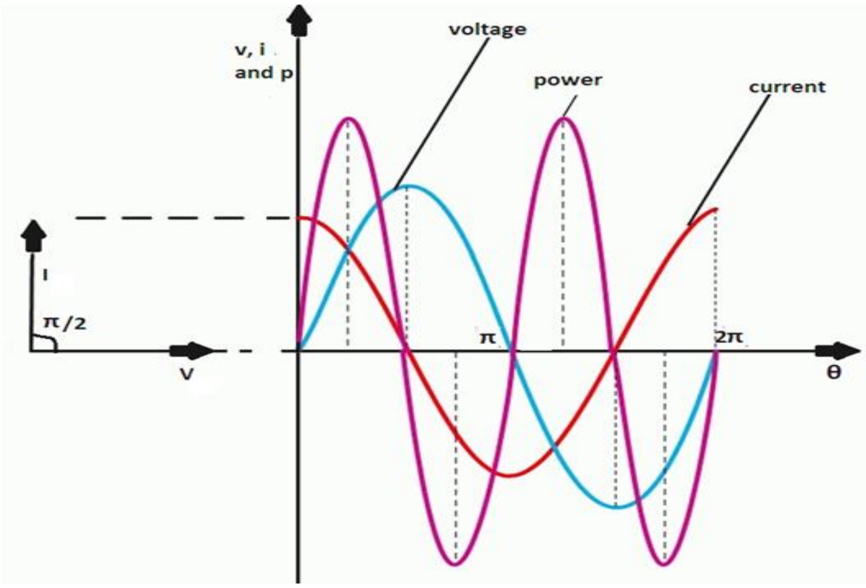
Where, $I_m = \frac{V_m}{X_C}$

Capacitve Reactance, $X_C = 1/\,2\pi f C$ ohms

Power absorbed by circuit = 0

Power factor, $\cos\phi = 0$

From the expression of instantaneous applied voltage and instantaneous current flowing through the purely capacitive circuit, it is observed that the current leads the voltage by π/2.

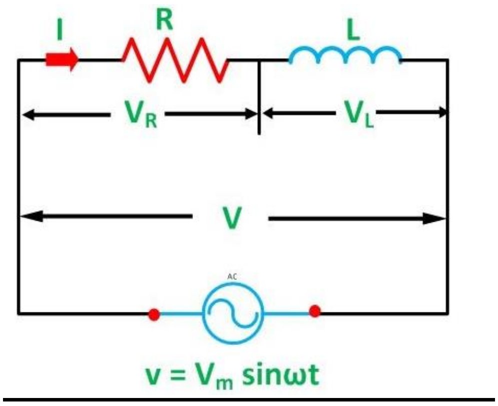


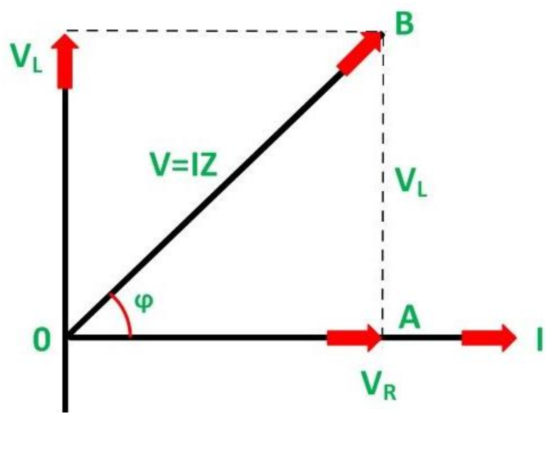
The power curve for the purely capacitive circuit is shown in the figure. It is clear that average power in a half cycle is zero as the negative and positive loop area under power curve is the same.

In the purely capacitive circuit, during the first quarter cycle, what so ever energy (or power) is supplied by the source is stored in the electric field set-up between the capacitor plates. In the next quarter cycle, the electric field collapses and the energy (or power) stored in the electric field is returned to the source. This process is repeated in every alternation. Hence, no power is consumed in the purely capacitive circuit.

AC through RL series circuit

When an AC supply voltage V is applied the current, I flows in the circuit. I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other.





Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L

- V – Total voltage of the circuit

$V_R = I R$ = RMS value of voltage across resistor

$V_L = IX_L$ = RMS value of voltage across inductor

where $X_L = 2\pi fL\ \Omega$= inductive reactance

From the phasor diagram:

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = L = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

From the phasor diagram it is clear that current lags behind the voltage by an angle (ϕ) less than 90^o.

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

The equation for current is $i = I_m \sin(\omega t - \varphi)$

The instantaneous power is given by the equation

P = vi

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below

$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s}I_{r.m.s}\cos\varphi = V\,I\cos\varphi$$

Where cosϕ is called the power factor of the circuit.

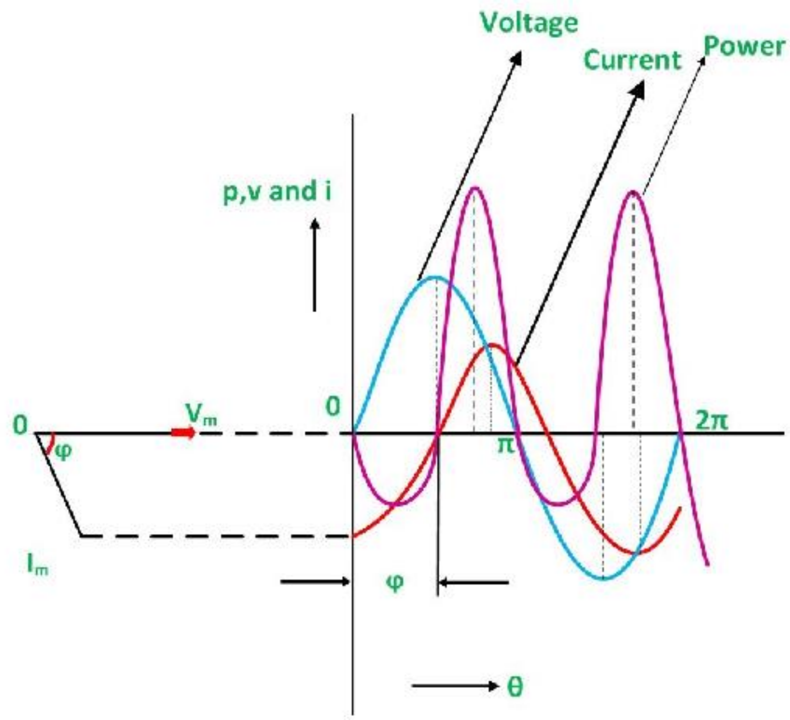
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Putting the value of V and cosϕ from the above equation the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R$$

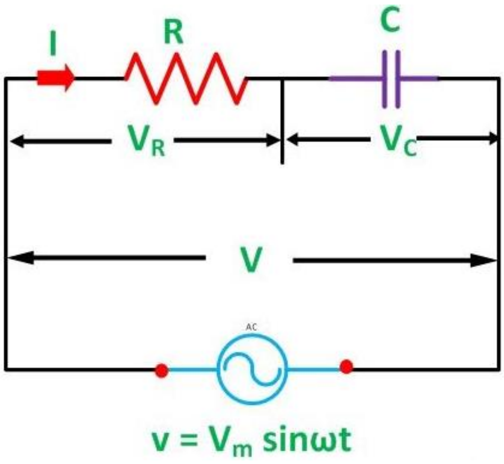
From the above equation it can be concluded that the inductor does not consume any power in the circuit.

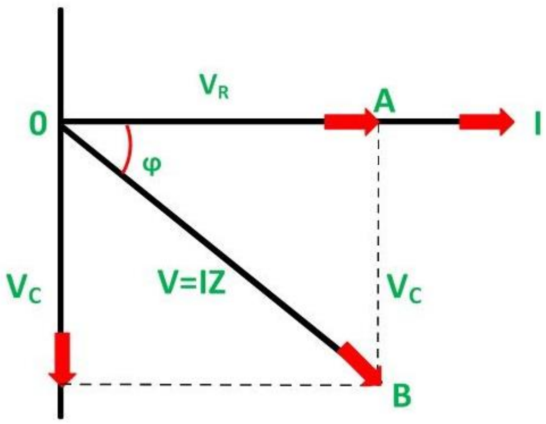
The **waveform** and **power curve** of the RL Series Circuit is shown below



RC Series Circuit

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied to and current I flows through the resistance (R) and the capacitance (C) of the circuit.The RC Series circuit is shown in the figure below





Where,

- V_R – voltage across the resistance R
- V_C – voltage across the capacitor C
- V – total voltage across the RC Series circuit

Now V_R = I R= RMS value of voltage across resistor
$V_L = IX_L$ = RMS value of voltage across inductor

where $X_L = 2\pi fL$ Ω= inductive reactance

From the phasor diagram of a RC series circuit

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where $$Z = \sqrt{R^2 + X_C^2}$$

Z is called impedance of the circuit. It is measured in ohms (Ω).

From the phasor diagram shown above it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the phase angle.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Thus the expression of instantaneous value of current through RC series circuit is given by

$$i = I_m \sin(\omega t + \varphi)$$

The instantaneous power is given by the equation

P = vi

By putting the value of v and i

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi)$$

The Average power consumed in the circuit over a complete cycle is given by

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

Where, $\cos\phi$ is called the power factor of the circuit.

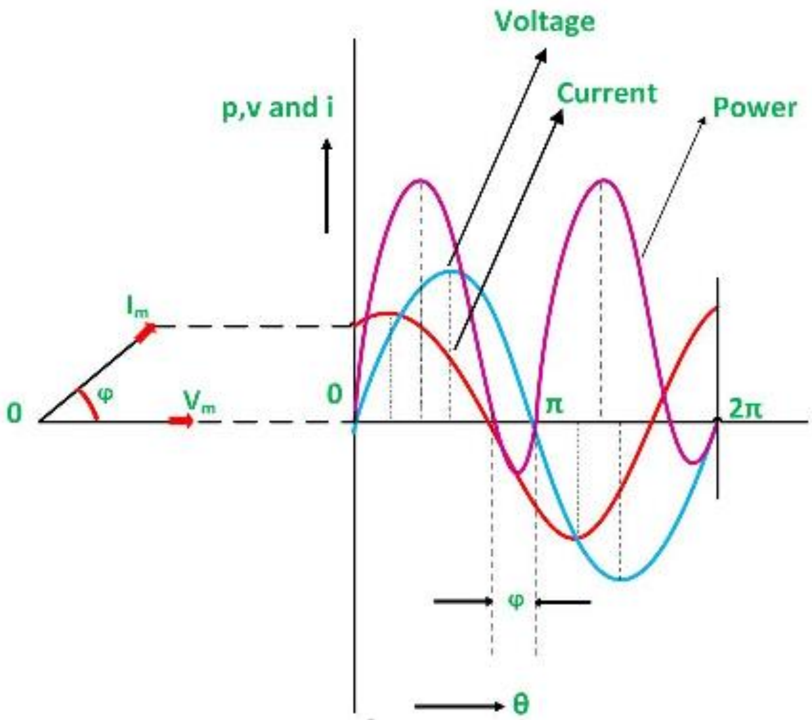
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Putting the value of V and $\cos\phi$ from the above equation the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R$$

From the above equation it is clear that the power is actually consumed by the resistance only and the capacitor does not consumes any power in the circuit.

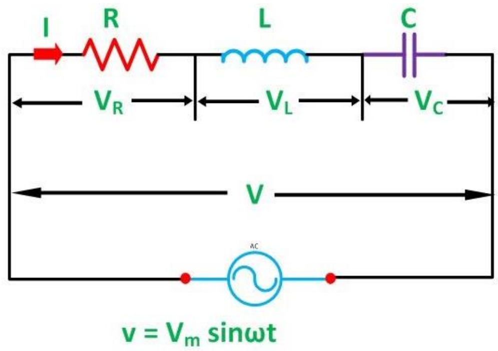
The waveform and power curve of the RC Circuit is shown below



The power is negative between the angle (180◦ – ϕ) and 180◦ and between (360◦ -ϕ) and 360◦ and in the rest of the cycle the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is positive.

RLC Series Circuit

The **RLC Series Circuit** is defined as when a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other. As all the three elements are connected in series so, the current flowing in each element of the circuit will be same. RLC series circuit is shown below:



In the RLC Series Circuit

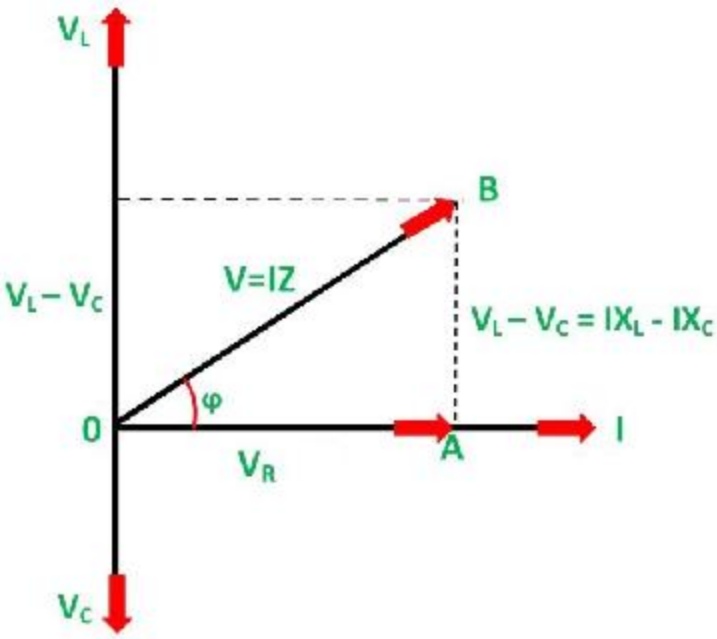
$X_L = 2\pi fL$ and $X_C = 1/2\pi fC$

When the AC voltage is applied through the RLC Series Circuit the resulting current I flows through the circuit, and thus the voltage across each element will be

- $V_R = IR$ that is the voltage across the resistance R and is in phase with the current I.

- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across the capacitor C and it lags the current I by an angle of 90 degrees.

The phasor diagram of the RLC Series Circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.



Hence from phasor diagram

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

The product of voltage and current is defined as power (since power only consumed by resister)

$$P = VI\cos\varphi = I^2R$$

Where cosϕ is the power factor of the circuit and is expressed as

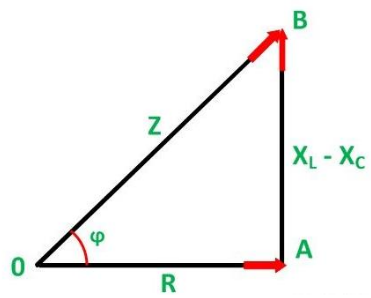
$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as a RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of power factor is unity.

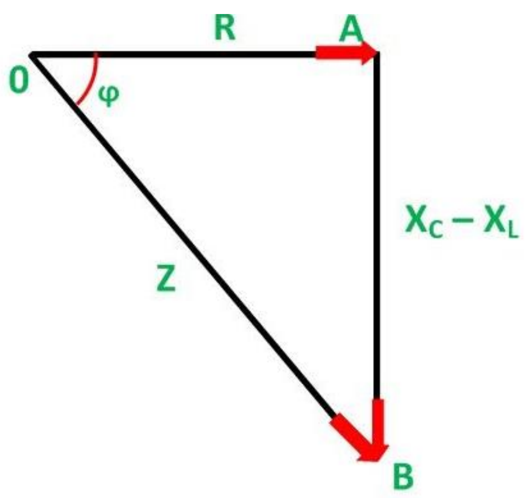
Impedance Triangle of RLC Series Circuit

The impedance triangle of the RLC series circuit, when ($X_L > X_C$) is shown below



If the inductive reactance is greater than the capacitive reactance then the circuit reactance is inductive giving a lagging phase angle.

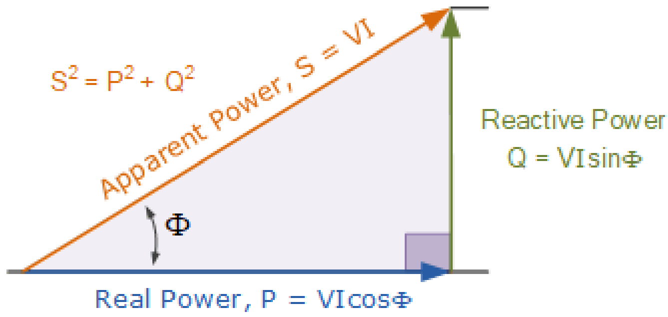
The impedance triangle of the RLC series circuit, when ($X_L< X_C$) is shown below:



When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as a capacitive and the phase angle will be leading.

Power Triangle :

Power triangle for RLC series circuit is shown below when ($X_L > X_C$)



Where:

P = I^2R or Real power that performs work measured in watts (W).

Q = I^2X or Reactive power measured in volt-amperes reactive(VAr)

S = I^2Z or Apparent power measured in volt-amperes (VA)

Φ is the phase angle in degrees. The larger the phase angle, the greater the reactive power

Cos(Φ) = P/S = W/VA = power factor, p.f.

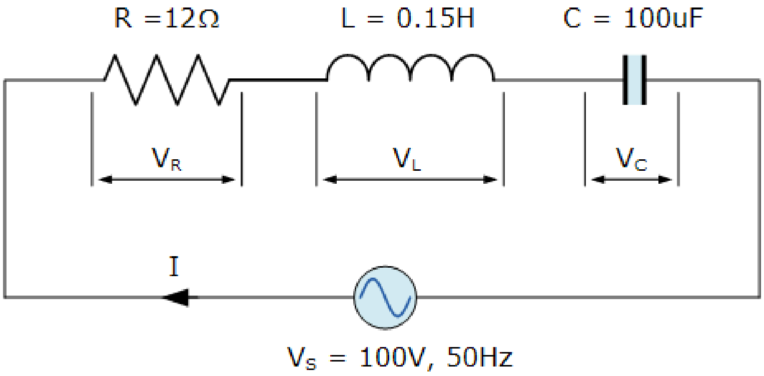
Sin(Φ) = Q/S = VAr/VA

Tan(Φ) = Q/P = VAr/W

Hence the power factor is calculated as the ratio of the real power to the apparent power.

RLC series circuit Example No1

A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uF are connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram.



Solution: Inductive reactance: $X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$

Capacitive Reactance:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$$

Circuit Impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current:

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\text{Amps}$$

Voltages across the Series RLC Circuit, V_R, V_L, V_C.

$$V_R = I \times R = 5.14 \times 12 = 61.7\text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2\text{ volts}$$

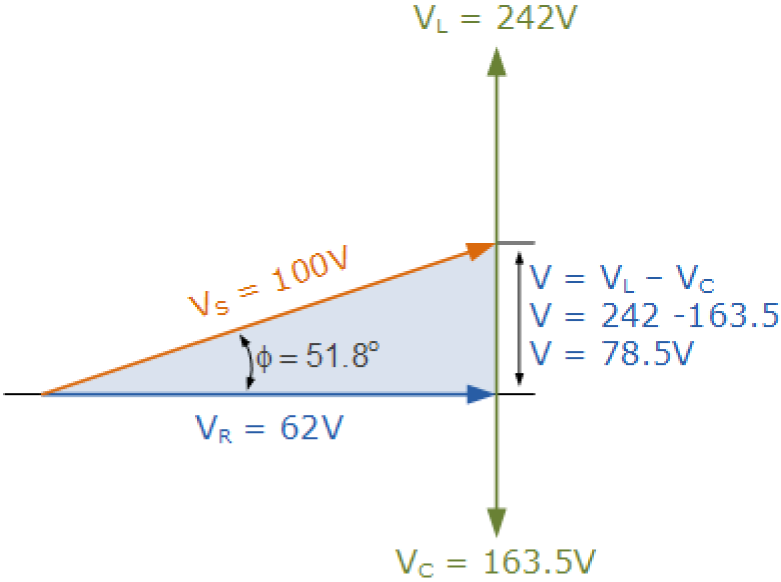
$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5\text{ volts}$$

Circuits Power factor and Phase Angle, θ.

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^{\circ} \text{ lagging}$$

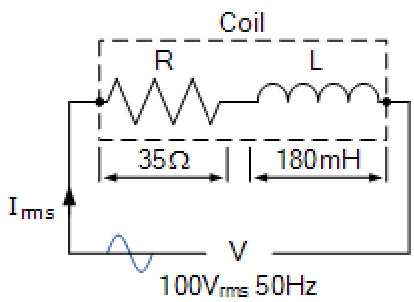
Phasor Diagram.



Since the phase angle θ is calculated as a positive value of 51.8° the overall reactance of the circuit must be inductive. Hence the current "lags" the source voltage by 51.8°

RLC series circuit Example No2

A wound coil that has an inductance of 180mH and a resistance of 35Ω is connected to a 100V 50Hz supply. Calculate: a) the impedance of the coil, b) the current, c) the power factor, and d) the apparent power consumed.



Also draw the resulting power triangle for the above coil.

Data given: R = 35Ω, L = 180mH, V = 100V and f = 50Hz.

(a) Impedance (Z) of the coil:

$$R = 35\Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.18 = 56.6\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{35^2 + 56.6^2} = 66.5\Omega$$

(b) Current (I) consumed by the coil:

$$V = I \times Z$$

$$\therefore I = \frac{V}{Z} = \frac{100}{66.5} = 1.5\ A_{(rms)}$$

(c) The power factor and phase angle, Φ:

$$\cos\phi = \frac{R}{Z}, \quad \text{or} \quad \sin\phi = \frac{X_L}{Z}, \quad \text{or} \quad \tan\phi = \frac{X_L}{R}$$

$$\therefore \cos\phi = \frac{R}{Z} = \frac{35}{66.5} = 0.5263$$

$$\cos^{-1}(0.5263) = 58.2^{o} \ (\text{lagging})$$

(d) Apparent power (S) consumed by the coil:

$$P = V \times I\cos\phi = 100 \times 1.5 \times \cos\left(58.2^{o}\right) = 79W$$

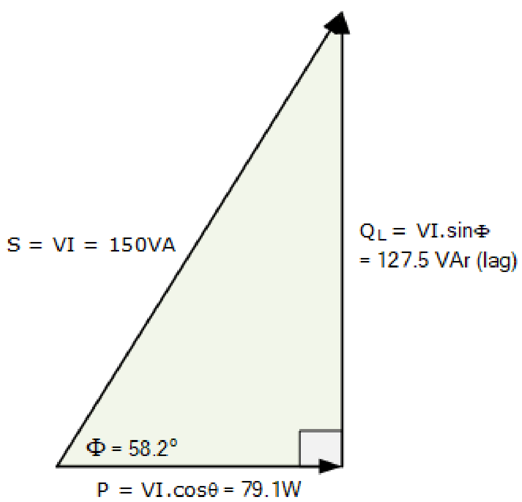
$$Q = V \times I\sin\phi = 100 \times 1.5 \times \sin\left(58.2^{o}\right) = 127.5VAr$$

$$S = V \times I = 100 \times 1.5 = 150VA$$

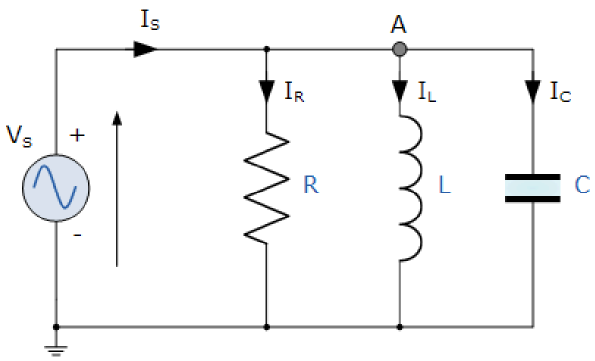
$$\text{or} \quad S^2 = P^2 + Q^2$$

$$\therefore S = \sqrt{P^2 + Q^2} = \sqrt{79^2 + 127.5^2} = 150VA$$

(e) Power triangle for the coil:



Parallel RLC Circuit



All the variable shown in the diagram are RMS quantities.

Here:

$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

$$\therefore\ I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$\text{where:}\quad I_R = \frac{V}{R},\quad I_L = \frac{V}{X_L},\quad I_C = \frac{V}{X_C}$$

Impedance of a Parallel RLC Circuit

$$R = \frac{V}{I_R} \qquad X_L = \frac{V}{I_L} \qquad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore\ \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Where

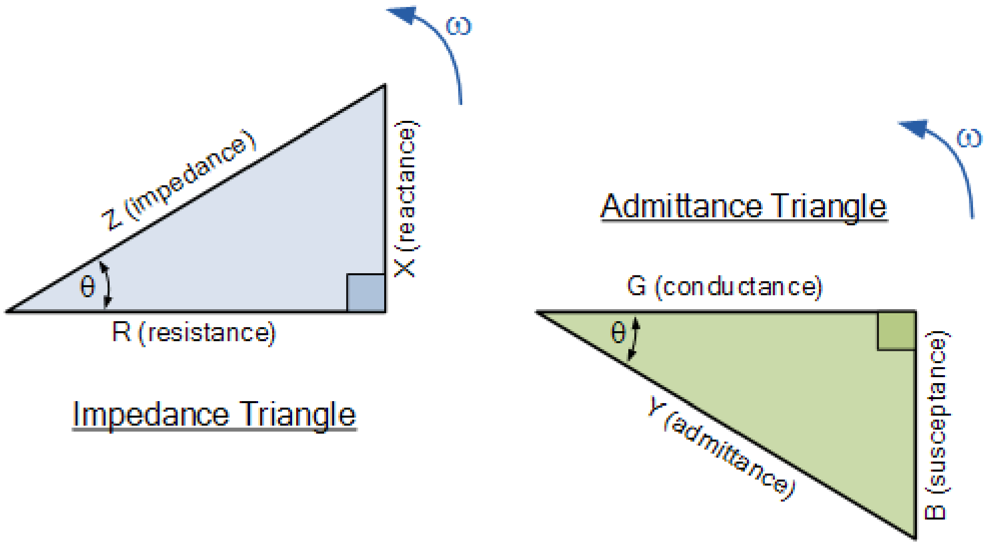
$\frac{1}{Z}$ = Y= admittance, $\frac{1}{R}$ = G= conductance, $\frac{1}{X_L}$= B_L = inductive susceptance, $\frac{1}{X_C}$= B_C = capacitive susceptance.

$$Y = \sqrt{G^2 + (B_L - B_C)^2}$$

where: $Y = \frac{1}{Z}$ $\quad G = \frac{1}{R}$

$$B_L = \frac{1}{\omega L} \qquad B_C = \omega C$$

Admittance Triangle for a Parallel RLC Circuit is shown below:



Giving us a power factor angle of:

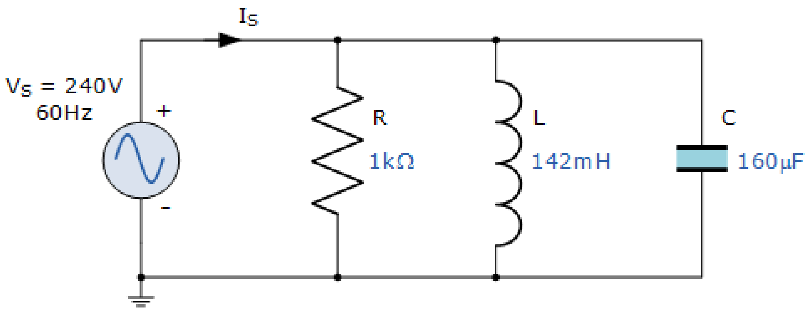
$$\cos\phi = \frac{G}{Y} \qquad \phi = \cos^{-1}\left(\frac{G}{Y}\right)$$

or

$$\tan\phi = \frac{B}{G} \qquad \phi = \tan^{-1}\left(\frac{B}{G}\right)$$

Parallel RLC Circuit Example No1

A 1kΩ resistor, a 142mH coil and a 160uF capacitor are all connected in parallel across a 240V, 60Hz supply. Calculate the impedance of the parallel RLC circuit and the current drawn from the supply.



Solution: R = 1kΩ

Inductive Reactance, (X_L):

$$X_L = \omega L = 2\pi f L = 2\pi.60.142\times10^{-3} = 53.54\Omega$$

Capacitive Reactance, (X_C):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.60.160\times10^{-6}} = 16.58\Omega$$

Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{53.54} - \frac{1}{16.58}\right)^2}}$$

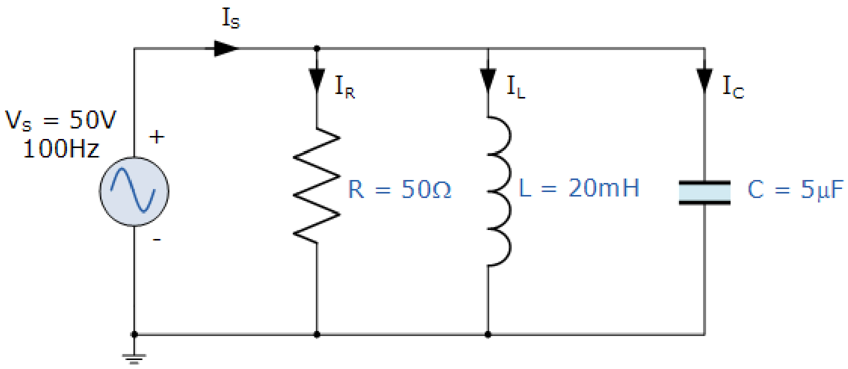
$$Z = \frac{1}{\sqrt{1.0\times10^{-6} + 1.734\times10^{-3}}} = \frac{1}{0.0417} = 24.0\Omega$$

Supply Current, (Is):

$$I_S = \frac{V_S}{Z} = \frac{240}{24} = 10 \text{ Amperes}$$

Parallel RLC Circuit Example No2

A 50Ω resistor, a 20mH coil and a 5uF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle. Also construct the current and admittance triangles representing the circuit.



1). Inductive Reactance, (X_L):

$$X_L = \omega L = 2\pi f L = 2\pi.100.0.02 = 12.6\Omega$$

2). Capacitive Reactance, (X_C):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.100.5\times10^{-6}} = 318.3\Omega$$

3). Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.3} - \frac{1}{12.6}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0058}} = \frac{1}{0.0788} = 12.7\Omega$$

4). Current through resistance, R (I_R):

$$I_R = \frac{V}{R} = \frac{50}{50} = 1.0\,(A)$$

5). Current through inductor, L (I_L):

$$I_L = \frac{V}{X_L} = \frac{50}{12.6} = 3.9\,(A)$$

6). Current through capacitor, C (I_C):

$$I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16\,(A)$$

7). Total supply current, (I_S):

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2} = \sqrt{1^2 + \left(3.9 - 0.16\right)^2} = 3.87\,(A)$$

8). Conductance, (G):

$$G = \frac{1}{R} = \frac{1}{50} = 0.02S \text{ or } 20mS$$

9). Inductive Susceptance, (B_L):

$$B_L = \frac{1}{X_L} = \frac{1}{12.6} = 0.08S \text{ or } 80mS$$

10). Capacitive Susceptance, (B_C):

$$B_C = \frac{1}{X_C} = \frac{1}{318.3} = 0.003S \text{ or } 3mS$$

11). Admittance, (Y):

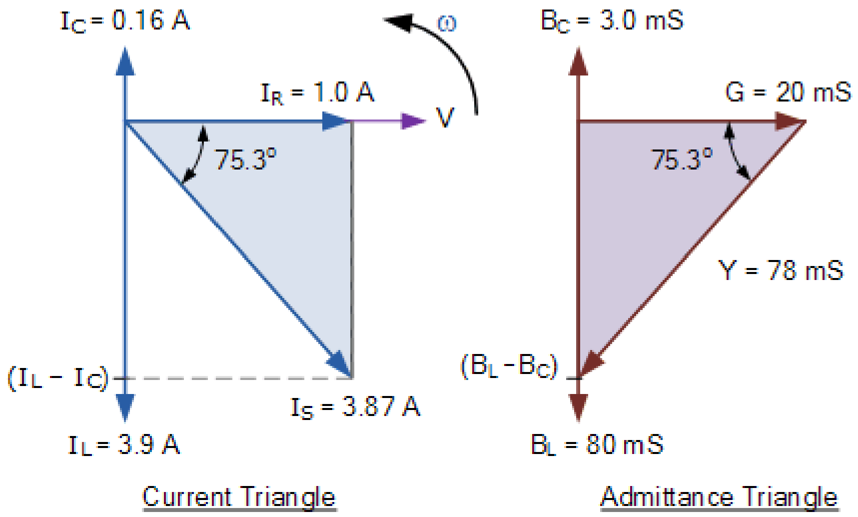
$$Y = \frac{1}{Z} = \frac{1}{12.7} = 0.078S \text{ or } 78mS$$

12). Phase Angle, (φ) between the resultant current and the supply voltage:

$$\cos\phi = \frac{G}{Y} = \frac{20mS}{78mS} = 0.256$$

$$\phi = \cos^{-1} 0.256 = 75.3^{\circ} \quad (\text{lag})$$

Current and Admittance Triangles



Q- Factor: Quality factor is defined as the reciprocal of power factor i.e

$$\text{Q-factor} = \frac{1}{\cos\phi}$$