MECHANICS

1. FUNDAMENTALS OF ENGINEERING MECHANICS

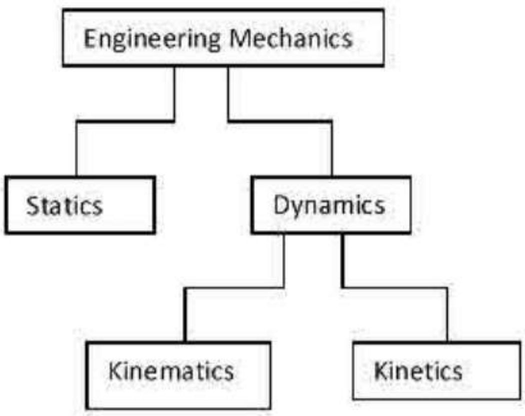
1.1 Fundamentals

Mechanics is that branch of science which deals with the forces and their effects on bodies on which they act and as a result the body may either in rest or motion.

Mechanics is divided into two parts.

(i) **Statics:-** Statics deals with the forces acting on a body under which the body is in rest.

(ii) **Dynamics:-** Dynamics deals with the forces acting on a body under which it is in motion.



Further the Dynamics is divided into two parts

(a) **Kinematics:-** Kinematics deals with the motion of bodies in which the agents responsible for motion is not considered.

(b) **Kinetics:-** Kinetics deals with the motion of the bodies in which the agents responsible for motion is considered. It deals with the relationship between forces and the resulting motion of bodies on which they act.

RIGID BODY

A rigid body is one which does not change its shape and size under the effect of force acting over it. It differs from an elastic body in the sense that the later undergoes deformation under the effect of forces acting on it and return to its original shape and size on removal of the forces acting on the body. The rigidity of a body depends upon the fact that how far it undergoes deformation under the effect of forces acting on it.

In real sense no solid body is perfectly rigid because everybody changes its size and shape under the effect of forces acting on it. Actually the deformation in a rigid body is very small and is generally neglected.

DEFINATIONS OF SOME RELATED TERMS

Mass:- The amount of matter contained in a body is called its mass, and for most problems in mechanics, mass may be considered constant.

Weight:- The force with which a body is attracted towards the centre of earth by the gravitational pull (g) is called its weight.

The relation between Mass(M) and Weight(W) of abody is given by the equation W=M*g

The value of g = 9.81 m/s^2.

Length:- This term is applied to the linear dimensions of a straight or curved line. e.g. the diameter of a circle is the length of a straight line which divides the circle in to two equal parts;the circumference is the length of its curved perimeter. Length is expressed in meter, cm,Km,feet etc.

Time:- The interval between two events is called time.It is expressed in Second,Minute and Hour.

Scalar:- Any physical quantity which has magnitude only is known as ScalarQuantity. Example:- Mass, distance,Volume, Time density etc.

Vector:- Any physical quantity which has magnitude as well as direction is known as Vector quantity. Example:- Force, Velocity, Displacement, Accelaration , Moment etc.

Fundamental units:- The basic quantities or fundamental quantities of mechanics are those quantities which cannot be expressed in terms of one another. Mass,Length,Time are usually considered as basic or fundamental quantities. The units of these quantities are called fundamental units and are denoted by M,L,T respectively.

Derived units:- The units of all other quantities expect the fundamental Quantities are derived with the help of fundamental units and thus they are known as derived units. For Example units of Velocity, acceleration, Density etc are derived units and are as follows.

Velocity(V) = Displacement/Time=L/T=LT^{-1}.

Acceleration(a) = Velocity/Time= $LT^{-1}/T = LT^{-2}$.

Density(ρ) = Mass/Volume = $M/L^3 = ML^{-3}$.

SYSTEM OF UNITS:

Generally we use four system of units and they are as follows

1. Foot –Pound-Second(FPS) System:- In this system the units of fundamental quantities i.e Length, Mass and Time are expressed in Foot, Pound and Second respectively.
2. Centimeter- Gram-Second(CGS) System:- In this system the value of Length, Mass and Time are expressed as Centimeter,Gram and Second.
3. Meter-Kilogram-Second(MKS) System:- In this system units of Length, Mass and Time are expressed in Meter, Kilogram and second respectively.
4. International System of units(S I Unit):- This system considers thre more fundamental units of Electric Current, Temperature and Luminous intensity in addition to the fundamental units of Mass,Length and Time.

FORCE

Defination of force and its units:-

Force:- Force is something which changes or tends to change the state of rest or of uniform motion of a body in a straight line. Force is the direct or indirect action of one body on another.It is a vector quantity.

There are different kinds of forces such as Gravitational, Frictional, Magnetic, Inertia or those caused by Mass and acceleration.

Units of Force:-

Absolute units:- Because the mass and acceleration are measured differently in different systems of units, so the units of force are also different in the various systems as below

FPS System:- Ft/S^2.(Poundal)

CGS System:- Cm/S^2.(Dyne)

MKS or S I System :- M/S^2.(Newton)

1 Newton = 10^5 Dynes.

Gravitational Units:- These are the units which are used by engineers for all practical purposes, these units depends upon the weight of a body. Now the weight of the body of mass (m) = mg, where g= Acceleration due to gravity.

So the gravitational units of force in the three systems of units i.e, FPS,CGS,and MKS are Pound weight, Gram weight and Kilogram weight.

The relationship of units of force is as follows

1 lb wt. = g Poundal = 32.2 Poundals

1 gm wt. = g Dynes = 981 Dynes

1 Kg wt. = g Netwon = 9.81Newtons.

Gravitational units of force= 'g' times the corresponding absolute units of force.

Representation of Force by Vector:-

Vector Representation:- A Force can be represented graphically by a vector as shown in Fig-1 and Fig-2.

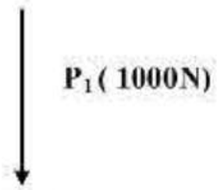


Fig-1

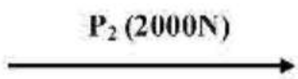


Fig-2

CHARACTERISTICS OF FORCE:-

The characteristics or elements of the Force are the quantities by which a force is fully represented. These are (i) Magnitude(i.e, 500 N, 1000N etc) (ii) Direction or line of action(Angle relative to a coordinate System) (iii) Sense or nature (Push or Pull) (iv) Point of application.

Effects of Force:-

When a force acts on a body, the effects produced in that body may be as follows:-

(i) It may bring a change in the motion of the body i.e, the motion may be accelerated or retarded.

(ii) It may balance the forces already acting on the body thus bringing the body to astste of rest or of equilibrium, and

(iii) It may change the size or shape of the body i.e, the body may be twisted , bend, stretched, compressed or otherwise distorted by the action of the force.

FORCE SYSTEMS

A force system is a collection of forces acting on a body in one or more planes.

According to the relative epositions of the lines of action of the forces, the forces may be classified as follows.

1. Coplanne Concurrent Colinear Force system:
   It is the simplest force system and includes those forces whose vectore lie along the same straight line .

Fig-3

2. Coplanner Concurrent Non- Parallel Force System:-
   Forces whose lines of action pass through a common point are called Concurrent forces. I

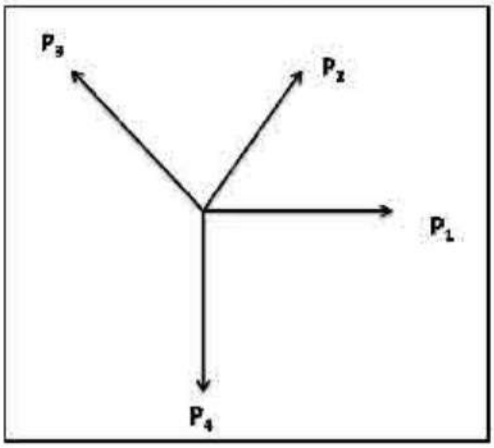


Fig-4

In this system lines of action of all the forces meet at a point but have different directions in the same plane are shown as in the figure.

3. **Coplanner Non- Concurrent Parallel Force system.**

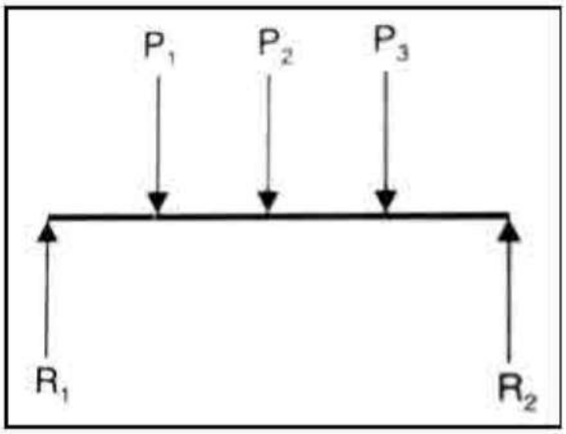


Fig-5

In this system, the lines of action of all the forces lie in the same plane and are parallel to each other but may not have same direction as shown in the figure.

4. **Coplanner Non- Concurrent Non- Parallel Force system:-**

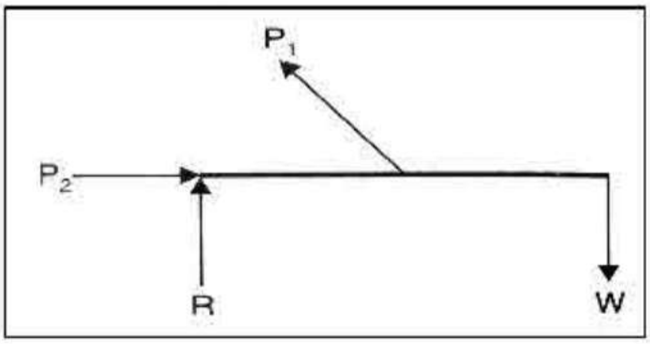


Fig-6

Such a system exists where the lines of action of all forces lie in the same plane but donot pass through a common point as shown in the figure.

5. **Non- Coplanner Concurrent Force System:-**

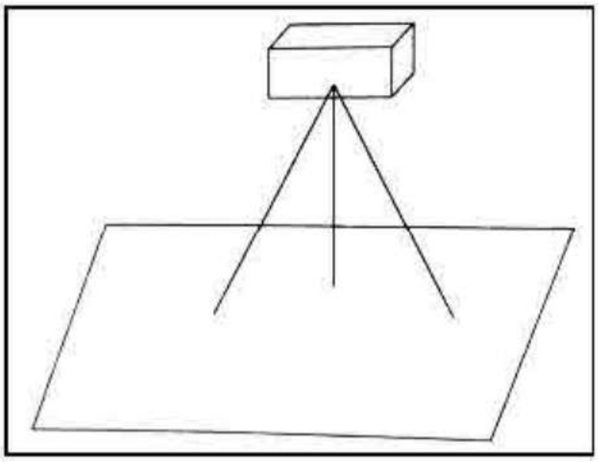


Fig-7

This system is evident where the lines of action of all forces donot lie in the same plane but do pass through a common point. An example of this force system is the forces in the legs of tripod support of a dumpy level.

6. **Non- Coplanner Non- Concurrent Force system:-**

Where the lines of action of all forces donot lie in the same plane and donot pass through a common point, a Non- Coplanner Non- Concurrent system is present.

Principle of transmissibility:-The Principle of Transmissibility of force states that when a force acts upon a body, its effect is the same whatever point in its line of action is taken as the point of application provided that the point is connected with the rest of the body in the same invariable manner.

Laws of Superposition:-

The action of a given system of force on a rigid body will in no way be changed if we add or subtract from them another system of forces in equilibrium.

Action & Reaction Forces:- Whenever there are two bodies in contact, each exerts a force on other.Outof these force one is called action and the other is called reaction. Action and reaction are equal and opposite.

FREE BODY DIAGRAM:-

A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces.A free body diagram is a process of isolating a body from all of its supports and in the place of the support a force of required magnitude is provided so that the position of the body will not change.

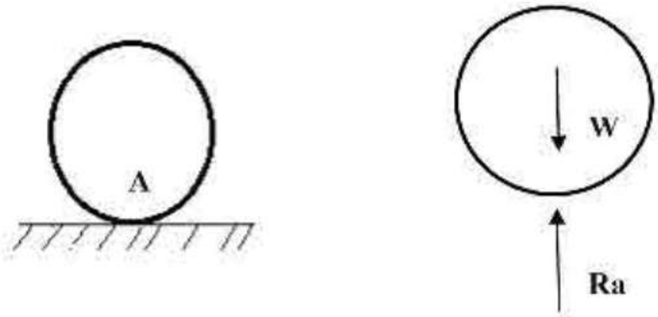


Fig-a

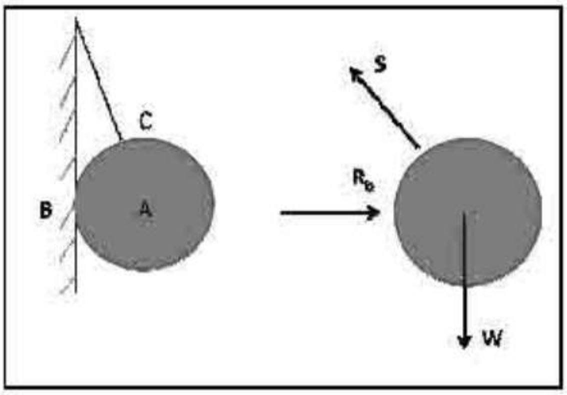


Fig-b

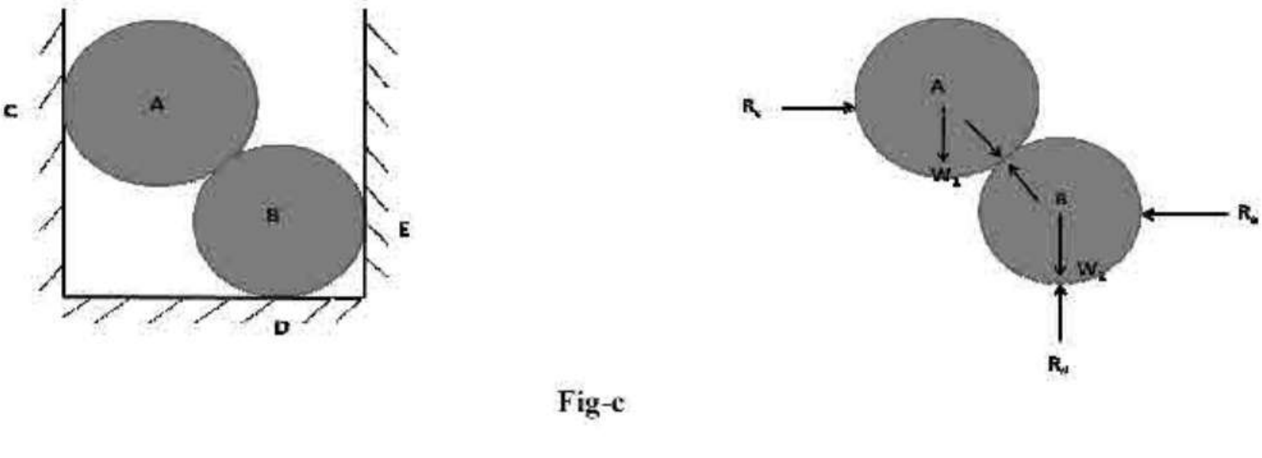


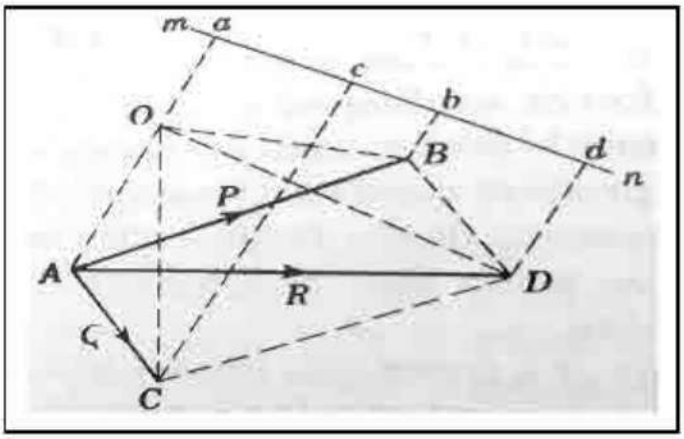
Fig-c

Theorm of Varignon:-

The moment of the resultant of two concurrent forces with respect to a center in their plane is equal to the algebraic sum of the moments of the components with respect to the same center.

Proof:-

Let us consider two forces 'P' and 'Q' with respect to the center 'O'.



In the plane of action of forces , we take any line *mn* perpendicular to the line OA joining the moment center with the point of concurrence of the forces and construct the perpendiculars A*a*,B*b*,C*c* and D*d* , as shown in figure.

Now the area of ΔOAB = $\frac{1}{2}$ OA. *ab,* the area of ΔOAC = $\frac{1}{2}$ OA. *ac,* and the area of ΔOAD = $\frac{1}{2}$ OA. *ad.*

Since *ad–ab+bd–ab+ac*

We conclude that Area ΔOAD = area ΔOAB+ area ΔOAC.

This proves the theorem.

1.3 RESOLUTION OF A FORCE

As two forces acting simultaneously on a particle acting along directions inclined to each other can be replaced by a single force which produces the same effect as that of the given force similarly a single force can be replaced by two forces acting in directions which will produce the same effect as that of the given force.

This breaking up of a force into two parts is called the resolution of a force.

RESULTANT OF A FORCE

A resultant force is a single force which can replace two or more forces and produce the same effect on the body as that of the forces.

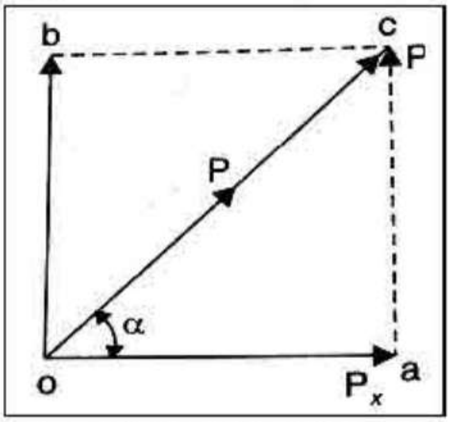
COMPONENT OF A FORCE

Generally a force is resolved into the following two types of components.

1. Mutually perpendicular components.
2. Non- Perpendicular components.

1. Mutually Perpendicular Components:-

Let us consider a force 'P' which is to be resolved is represented in both magnitude and direction by *'oc'* in the figure below.



Let P_x is the component of force P in the direction oa making an angle 'α' with the direction *oc* ₂

Complete the rectangle *oacb*.

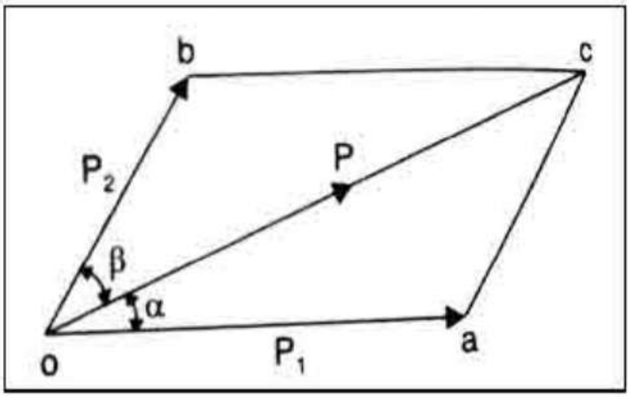
The other component P_y at right angle to P_x will be represented by 'ob' which is also equal to *ac*.

Now from the right angled triangle *oac*.

P_x= oa = P cosα and P_y = Psinα.

2. Non- Perpendicular Component.

Refering figure below Let oc represents the given force P in magnitude and direction to some scale.



Draw oa and ob making angle 'α' and 'β' with oc.

Through c draw ca parallel to 'ob' and 'cb' parallel to 'oa' to complete the parallelogram 'oacb'.

Now the vectors oa and ob represent in magnitude and direction as P_1 and P_2 respectivily.

Now from the triangle *'oac'*, by applying sine rule

$$\frac{oa}{sin\beta} = \frac{oc}{sin[180-(\alpha+\beta)]} = \frac{oc}{sin\alpha}$$

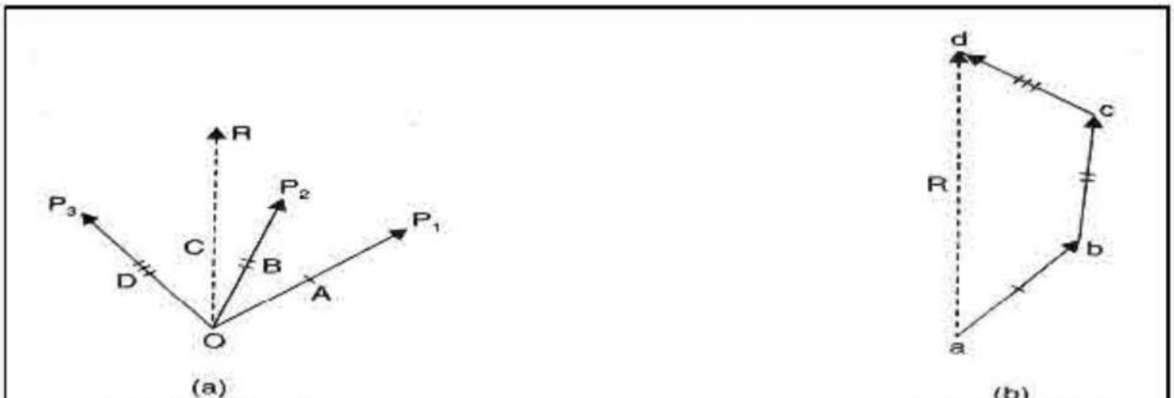
or $\frac{P_1}{sin\beta} = \frac{P}{sin(\alpha+\beta)} = \frac{P_2}{sin\alpha}$

so $P_1 = P.\frac{sin\beta}{sin(\alpha+\beta)}$ and $P_2 = P.\frac{sin\alpha}{sin(\alpha+\beta)}$

RESULTANT OF SEVERAL COPLANNER CONCURRENT FORCES:-

For the resultant of a number of concurrent forces any of the following two methods are used.

1. **Graphical Method**



(a) Space diagram (b) Vector diagram

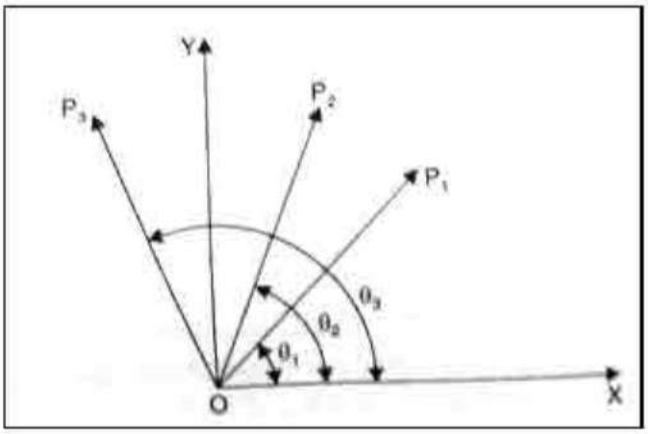
The figure above shows the forces P_1 P_2 and P_3 simultaniously acting at a point O.

Draw vector ab equal to force P_1 to some scale and parallel to the line of action of P_1.

From 'b' draw vector '*bc*' to represent force P_2 in magnitude and direction.

Now from 'c' draw vector 'cd' equal and parallel to P_3. Join 'ad' which gives the required resultant in magnitude and direction as per fig-(b) the vector diagram above.

2. **Analytical Method**



The resolution of P_1 in the direction OX is $P_1 Cos\theta_1$, and in the direction OY is OX is P_1 $Sin\theta_1$.

The resolution of P_2 in the direction OX is $P_2 Cos\theta_2$, and in the direction OY is OX is P_2 $Sin\theta_2$.

And the resolution of P_3 in the direction OX is P_3 Cosθ_3 , and in the direction OY is OX is P_3 Sinθ_3.

If the resultant R makes an angle θ with OX , then as per resolution,

R Cosθ = P_1Cosθ_1+ P_2Cosθ_2+ P_3 Cosθ_3 $= \sum H$

And R Sinθ = P_1 Sinθ_1+ P_2 Sinθ_2+ P_3 Sinθ_3 $= \sum V$

Now Resultant R = $\sqrt{(\sum H^2) + (\sum V^2)}$

Now $\frac{\text{R Sin}\theta}{\text{R Cos}\theta} = \frac{\sum V}{\sum H}$

So $\theta = tan^{-1}\left(\frac{\sum V}{\sum H}\right)$

Sign Convention for resolution:-

The upward forces (↑) is considered - Positive (+)

The downward forces (↓) is considered - Negetive (-)

The Right Hand Side forces is - Positive (+)

The left Handed Forces is- Negative (-)

Example -1

The following forces act a point:

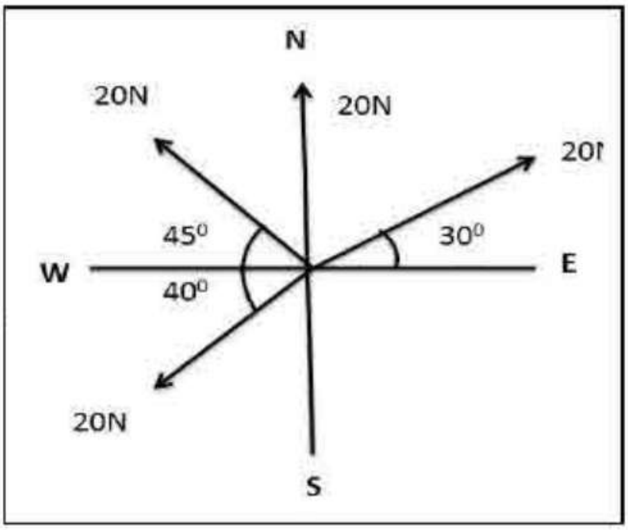
(i). 20N inclined at 30^0 towards north of east.

(ii). 25N towards north

(iii). 30N towards north west and

(iv). 35N inclined at 40^0 towards south of west.

Find the magnitude and direction of the resultant force.



Answer:-

Magnitude of the resultant force

Resolving all the forces horizontally i.e. along East -west line,

$\Sigma H = 20 \text{ Cos } 30^0 + 20 \text{ Cos } 90^0 + 30 \text{ Cos } 135^0 + 35 \text{ Cos } 220^0$ N

$= (20X0.866) + (25X0) + 30\ (-\ 0.707) + 35(-\ 0.766)$ N

$= -\ 30.7$ N

And now resolving all the forces vertically i.e. along north-south line,

$\Sigma V = 20 \text{ Sin } 30^0 + 25 \text{ Sin } 30^0 + 30 \text{ Sin } 135^0 + 35 \text{ Sin } 220^0$ N

$= (20X0.5) + (25\ X\ 1.00) + (30\ X\ 0.707) + 35\ (-\ 0.6428)$ N

$= 33.7$ N

We know that the magnitude of the resultant force,

$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2}$ N

$= 45.6$ N ………………. (Ans.)

<u>*Direction of the resultant force*</u>

Let 0 = Angle, Which the resultant force makes with the East.

Thus, Tan $\theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -\ 1.098$ or $\theta = 47^0 42'$

Since $\sum H$ is – Ve and $\sum V$ is +Ve , therefore θ lies between 90^0 and 180^0 .

Thus , Actual $\theta = 180^0 - 47^0 42' = 132^0 18'$ ……….(Ans.)

Example-2

Determine the magnitude and direction of the resultant of the two forces of magnitude 12 N and 9 N acting at a point, if the angle between the two forces is 30^0.

Given:-

$$F_1 = 12N \qquad F_2 = 9N \qquad \alpha = 30^0$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$
$$R = \sqrt{12^2 + 9^2 + 2\times12\times9\times\cos30^0}$$
$$R = 20.3N$$
$$\theta = \tan^{-1}\left(\frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}\right)$$
$$\theta = \tan^{-1}\left(\frac{9\sin30^0}{12+9\cos30^0}\right)$$
$$\theta = 12.81^0$$

Example-3

2.Find the magnitude of two equal forces acting at a point with an angle of 60^0 between them, if the resultant is equal to $30\sqrt{3}N$

GIVEN:

$$F_1 = F_2 = F, say$$

$$R = 30\sqrt{3}N, \alpha = 60^0$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$
$$R = \sqrt{F^2 + F^2 + 2F\times F\times\cos60^0}$$
$$R = \sqrt{F^2 + F^2 + F^2}$$
$$R = \sqrt{3}F$$
$$F = 30N$$