Chapter – 2

Equilibrium of Forces

If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium.

The force, which bring the set of forces in equilibrium is called on equilibrient. The equilibriant is equal to the resultant force in magnitude, but opposite in direction.

Analytical conditions of equilibrium of a co-planner system of concurrent forces.

We know that the resultant of a system of co-planner concurrent forces is given by :

$R=\sqrt{(\Sigma x)^2+(\Sigma y)^2}, where$ $\qquad \therefore \quad \Sigma y=0$

Σx = Algebraic sum of resolved parts of the forces along a horizontal direction.

Σy = Algebraic sum of resolved parts of the forces along a vertical direction.

$R^2=(\Sigma x)^2+(\Sigma y)^2$

If the forces are in equilibrium, R=0

$0=(\Sigma x)^2+(\Sigma y)^2$

Sum of the squared of two quantities is zero when each quantity is separately equal to zero.

$\therefore \quad \Sigma x=0$ and $\Sigma y=0$

Hence necessary and sufficient conditions of a system of co-plan concurrent forces are :

i) The algebraic sum of the resolved parts of the forces is some assigned direction is equal to zero.

ii) The algebraic sum of the resolved parts of the forces is a direction nat right angles to the assigned direction is equal to zero.

Graphical conditions of equilibrium of a system of co-planner concurrent forces :

Let a number of forces acting at a point be in equilibrium. Then, it can be said that the resultant of these forces is nil. Hence the length of the closing line of the polygon drawn to represent the given forces taken in orders will be nil. In other words, the length of closing line of the vector diagram drawn with the given forces in orders, is nil. This means the end point of the vector diagram must coin side with the starting point of the diagram. Hence the vector diagram must be closed figure.

So, graphical condition of equilibrium of a system of co-planner concurrent forces may be stated as follows :

If a system of co-planner concurrent forces be in equilibrium, the vector diagram drawn with the given forces taken in orders, must be closed figure.

Lami's Theorem :

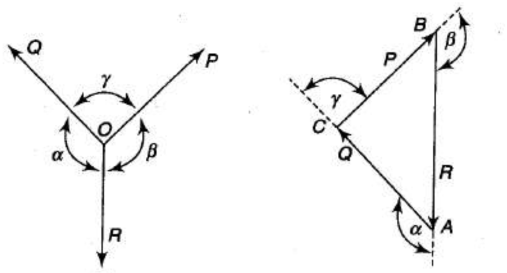
If there concurrent forces are acting on a body, kept in an equilibrium, then each force is force is proportional to the sine of the angle between the other two forces and the constant of proportionality is the same.

Consider forces, P,Q and R acting at a point 'O' mathematically, hom's theorem is given by the following equilibrium :

$$\frac{P}{Sin\alpha}=\frac{Q}{Sin\beta}=\frac{R}{Sin\gamma}=K$$

Since the forces are on equilibrium, the triangle of forces should close, con responding to the forces, P,Q and R acting at a point 'O'. The angle of triangle are

$\angle A=\lambda-\alpha$
$\angle B=\lambda-\beta$
$\angle C=\lambda-\gamma$



(a) (b)

From the rule for the triangle we get.

$$\frac{P}{Sin(\lambda-\alpha)}=\frac{Q}{Sin(\lambda-\beta)}=\frac{R}{Sin(\lambda-\gamma)}$$

$$Sin(\lambda-\alpha)=Sin\alpha$$

$$Sin(\lambda-\beta)=Sin\beta$$

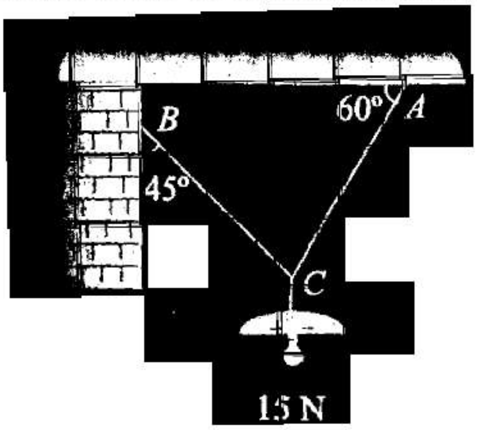
$$Sin(\lambda-\gamma)=Sin\gamma$$

Or we can write the equation (1) according to Lami's Theorem i.e.

$$\frac{P}{Sin\alpha}=\frac{Q}{Sin\beta}=\frac{R}{Sin\gamma}$$

Example :

An electric light fixture weighing 15 N hangs from a paint C, by two strings AC and BC. The string AC is inclined at 60^0 to the horizontal and BC at 45^0 to the vertical as shown in figure, using lami's theorem determine the forces in the strings AC and BC.

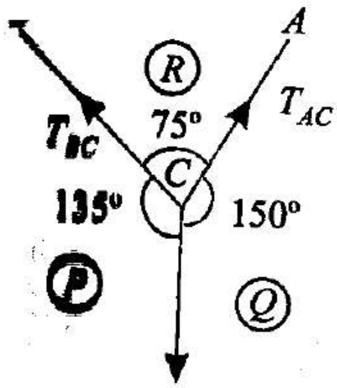


Solution :

Given weight at C = 15 N

T_{AC} = Force in the string AC.

T_{BC} = Force in the string BC.



According to the system of forces and the above figure, we find that angle between T_{AC} and ISN is 150^0 and angle between T_{BC} and 15 N is 135^0.

$$\angle ACB = 180^0 - (40^0 + 60^0) = 75^0$$

Applying Lami's Theorem :

$$\frac{15}{Sin75^0} = \frac{T_{AC}}{Sin135^0} = \frac{T_{BC}}{Sin150^0}$$

$$\therefore \; T_{AC} = \frac{15Sin.135^0}{Sin75^0} = 10.98N \text{ (Ans)}$$

and T_{BC} = $T_{BC} = \dfrac{15Sin.150^0}{Sin75^0} = 7.76N$ (Ans)

Body Diagram :

Free body diagram is a sketch of the isolated body, which shows the external forces on the body and the reaction extended on it by the removed elements.

The general procedure for constructing a free body diagram is as follows :

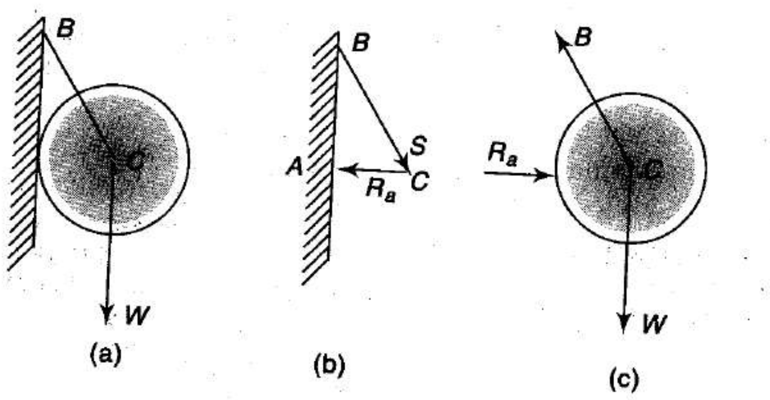
1. A sketch of the body is drawn, by removing the supporting surfaces.

2. Indicate on this sketch all the applied on active forces which send to set the body in motion, such as those caused by weight of the body on applied forces etc.

3. Also indicate on this sketch all the reactive forces, such as those caused by the constrains or supports that tend to prevent motion. (The sense of unknown reaction should be assumed. The correct sense will be determined by the solution of the problem. A positive result indicates that the assumed sense is correct A negative results indicates that the correct sense is opposite to the assumed sense).

4. All relevant dimensions and angles, reference axes are shown on the sketch.

Example :



CHAPTER – 3

FRICTION

When a body moves or trends to move over another body, an opposing force develops at the contact surface. This force opposes the movement is called frictional force or friction.

Frictional force is the resistance offered when a body moves over another body assist the motion.

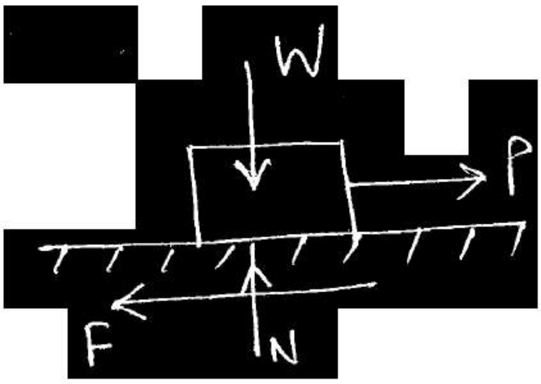
There is a limit beyond which the magnitude of this force cannot increase when applied force more than this limit there will be motion. When applied force less than this limit value, the body remains at rest and such frictional force is called static friction. When body moves (applied force more than limiting friction) the frictional resistance is known as Dynamic friction.

Dynamic friction is found less than limiting friction.

Dynamic friction < Sliding friction / Rolling friction

Coefficient of friction :

Experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces and this ratio is called coefficient of friction :



Co-efficient of friction = F/N = μ

F → Limiting friction (Frictional force)

N → Normal reaction

Laws of friction :

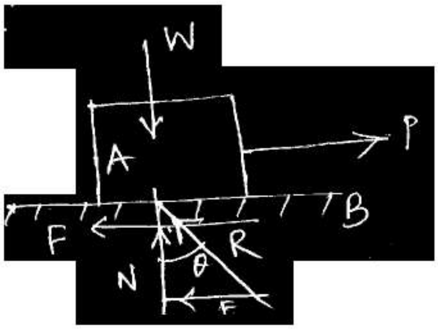
1) Force of friction is directly proportional to the normal reaction and always opposite in the direction of motion.

2) Force of friction depends upon the roughness/ smoothness of the surface.

3) Force of friction is independent of the areas of the contact.

4) Force of friction is independent of the sliding velocity.

Angle of friction :



Consider the block 'A' resting on horizontal plane 'B'.

Let, P → horizontal force applied.

F → Frictional force

N → Normal reaction.

Let R be the resultant reaction between normal reaction and force of friction acts at angle θ to the normal Reaction. θ → Angle of friction.

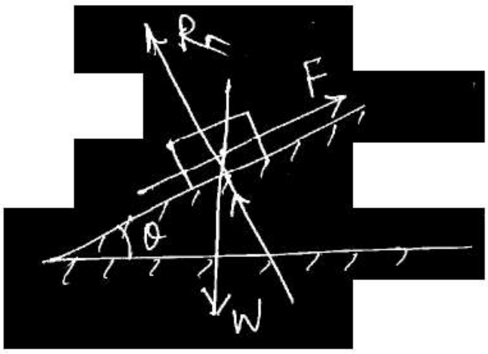
Tan θ = F/N

As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value

Tan α = F/N = μ

A → Angle of friction. (Angle of limiting friction)

Angle of Repose :



It is the maximum inclined plane with the horizontal for which a body lying on the inclined plane will be on the point of sliding down.

Angle of Repose is equal to the angle of friction .

Consider a block of weight 'W' resting on a rough inclined plane (angle)

Rn → Normal reaction

f → Frictional force

F= W Sin α , Rn = W Cos α

Tan α = $\frac{E}{Rn}$

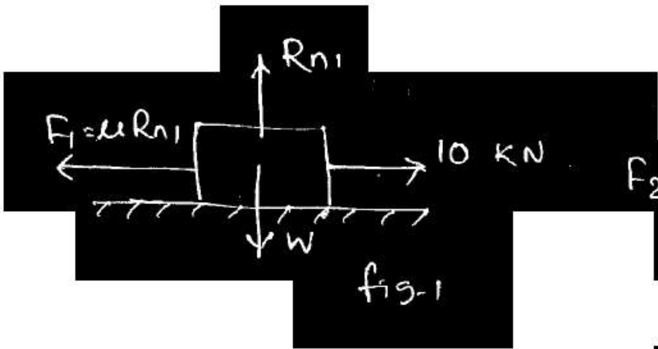
But we knew that $\tan \phi = \frac{E}{Rn} = \mu$

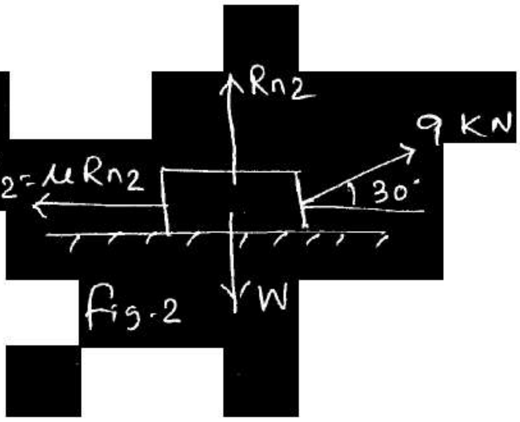
$\text{Tan } \alpha = \text{Tan } \phi$ $\boxed{\alpha = \phi}$

<u>Ex – 1</u>

A body resting on a horizontal plane can be moved slowly along the plane by a horizontal force of 10 KN. A force of a 9 KN inclined at 30^0 to the horizontal direction will suffice to move the block along the same direction. Determine the co-efficient of friction and weight of the body.

<u>Solution :</u>





Let μ = Co-efficient of friction

W = Weight of the body.

From fig-1

$F_1 = \mu R_{n1} = 10$ ----------------------(1)

$R_{n1} = W$ ----------------------(2)

So $\mu W = 10$ --------------------------- (3)

$9 \cos 30^0 = F_2 = \mu R_{n2}$ -------------- (4)

$R_{n2} + 9 \sin 30 = W$ ------------------- (5)

$R_{n2} = W - 4.5$ -------------------------- (6)

Putting the value of R_{n2} in equation(4)

$\mu [W-4.5] = 9 \cos 30^0 = 7.794$

$\mu W - \mu 4.5 = 7.794$ ------------------ (7)

Putting the value of μW from (3)

$10 - \mu 4.5 = 7.794$

$\mu 4.5 = 10 - 7.794$ $\boxed{\mu = 0.49}$

Putting the value of μ in equation (3)

.49 W = 10 $\qquad$ **W = 20.408 KN.**

A body resting on rough horizontal plane, required a pull of 18 KN at 30^0 to the plane just to move it. It was also found that a push of 22 KN inclined at 30^0 to the plane just moved the body. Determine the weight of the body and co-efficient of friction.

Solution :

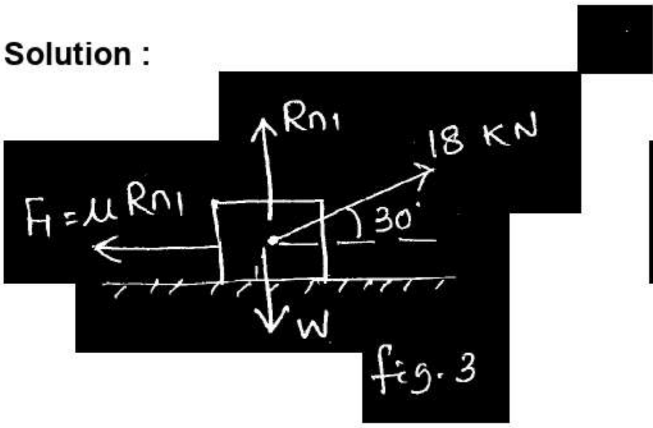


fig. 3

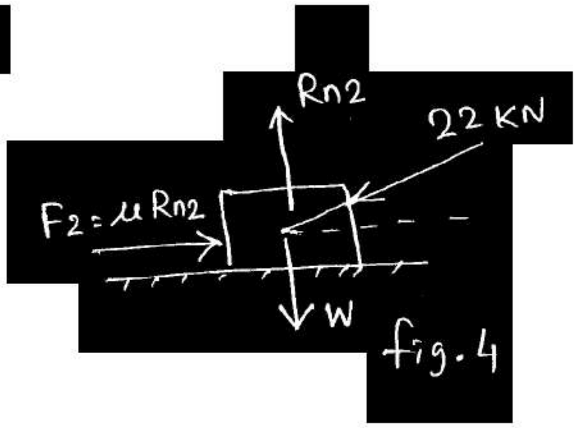


fig. 4

Let $\quad$ W → $\quad$ Weight of the body

μ → $\quad$ Co-efficient of friction

From fig - 3

$F_1 = \mu R_{n1} = 18 \cos 30^0 = 15.59$ KN -------------(1)

$R_N + 18 \sin 30^0 = W$

$R_N = W - 9$ -- (2)

Putting the value of equation (2) in equation (1)

μ (W-9) = 15.59 -------------------------(3)

From fig. 4.

$F_2 = \mu R_{n2} = 22 \cos 30 = 19.05$ ------------------(4)

$R_{n2} = W + 22 \sin 30^0 = W + 11$ --------------(5)

PUtting the value of equation (5) in equation (4)

μ (W+11) = 19.05 ----------------------------(6)

From equation (3) and equaiton (6)

$$\mu = \frac{15.59}{W-9}, \ \mu = \frac{19.05}{W+11}$$

$$\frac{15.59}{19.05} = \frac{W-9}{W+11}$$

W = 99.12 KN

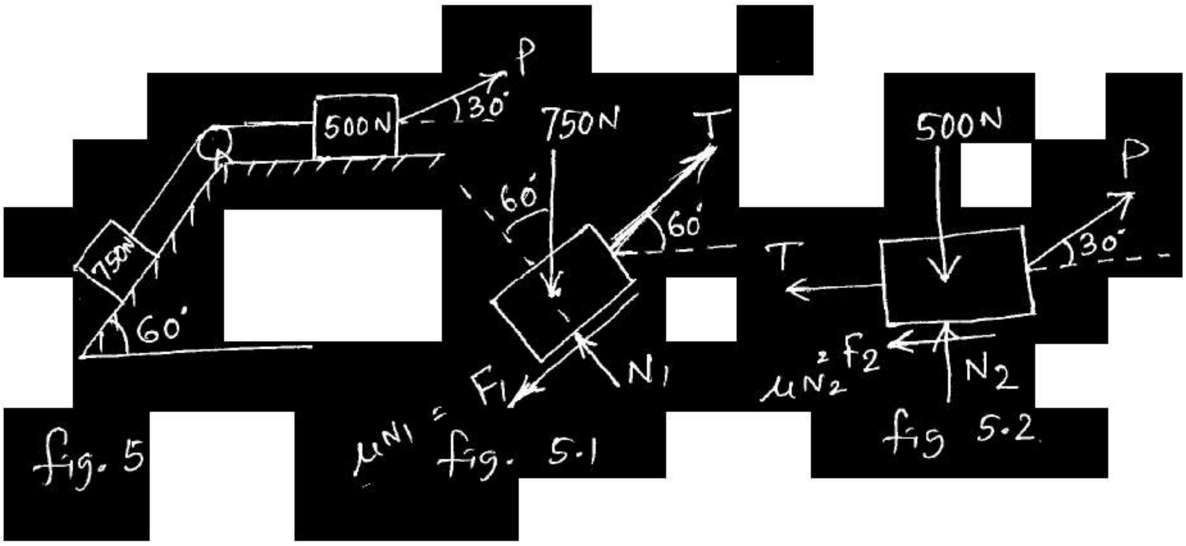
Putting the value of W in equation (3)

$$\mu(99.12-9)=15.59$$

$$\mu=\frac{15.59}{90.12}=0.173$$

Ex – 3

What is the value of P in the system shown in fig- 5 to cause the motion of 500 N block to the right side ? Assume the pulley is smooth and the co-efficient of friction between other contact surface is 0.20.



Solution :

Consider the FBD of fig. 5.1

$N_1 = 750 \text{ Cos } 60^0$

$N_1 = 375$

$750 \text{ Sin } 60 + \mu N_1 = T$

750 Sin 60 + 0.2 x 375 = T = 724.52 N --------------------(2)

Consider the FBD of fig. 5.2

$N_2 + P \text{ Sin } 30^0 = 500$

$N_2 = 500 - 0.5 P$ ---------------------------------(3)

$P \cos 30 = T + \mu N_2 = 724.52 + 0.2\ (500 - .5P)$

P Cos 30 + .1 P = 724.52 + 100

P (Cos 30 + 0.1) = 824.52

P = 853.52 N

Ladder friction :

It is a device for climbing up or down.

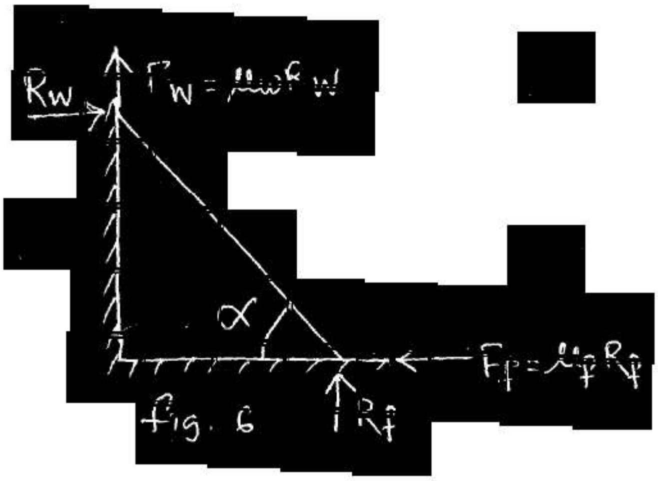


fig. 6

Since this system is in equilibrium, the algebraic sum of the horizontal and vertical component of the forces must be zero and the algebraic sum of the moment must also be zero.

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

Ex – 5

A ladder of length 4m, weighing 200N is placed against a vertical wall as shown in fig. 7. The co-efficient of friction between the wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. In addition to self weight, the ladder has to support a man weighing 600N at a distance of 3 m. from A calculate the minimum horizontal force to be calculate the minimum horizontal force to be applied at A to prevent slipping.

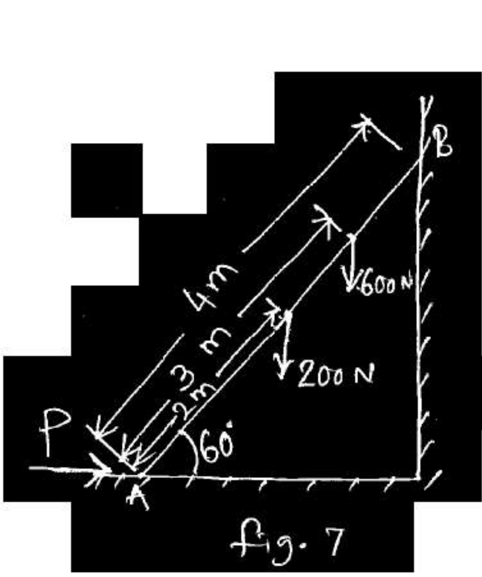


fig. 7

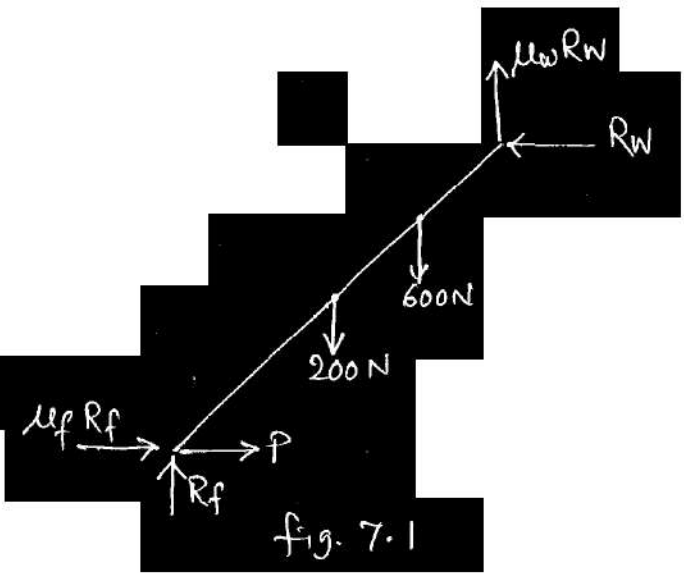


fig. 7.1

Solution :

The FBD of the ladder is shown in fig. 7.1

Taking moment at A

Rw x 4 Sin 60^0 + μ w Rw 4 Cos 60

= 600 x 3 cos 60 + 200 x 2 cos 60

866 Rw + 0.2 x 0.5 Rw = 275

0.966 Rw = 275

Rw = 284.68 -----------------------(1)

600 + 200 = Rf + μ wRw

Rf = 800 - 0.2 x 284 .68

= 743.06 ----------------------------------- (2)

P + μ f Rf = Rw

P = 284.68 - 0.3 x 743.06 = 61.76

Ex – 6

The ladder shown in fig. 8 is 6m. long and is supported by a horizontal floor and vertical wall. The co-efficient of friction between the floor and the ladder is 0.25 and between the wall and the ladder is 0.4 The self weight of the ladder is 200 N. The ladder also supports a vertical load of 900 N at C which is at a distance of 1 m from B. Determine the least of value of α at which the ladder may be placed without slipping.

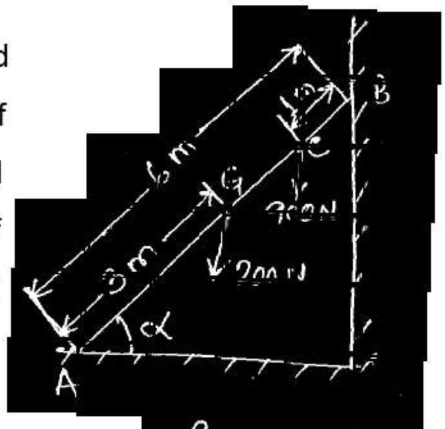


fig. 8

Solution :

μ w = Co-efficient of friction between wall and ladder = 0.4

μ f = Co-efficient of friction between floor and ladder = 0.25

The FBD of the ladder is shown in fig. 8.1

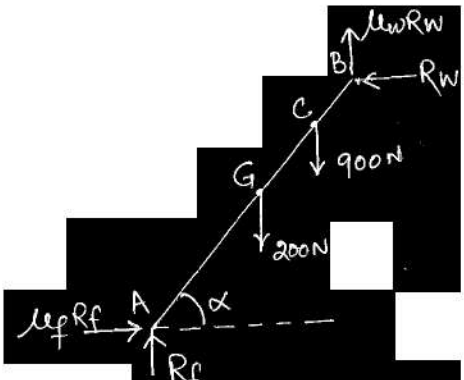


fig. 8.1

Rw = μ f Rf = 0.25 Rf ---------------- (1)

Rf + μ w Rw = 900 + 200 = 1100

Rf + 0.4 x .25 Rf = 1100

Rf = 100 N --------------------- (2)

So Rw = 250 N -------------------- (3)

Taking moment about A

Rw x 6 Sin α + μ w Rw x 6 Cos α

= 200 x 3 Cos α + 900 x 5 Cos α

250 x 6 Sin α + 0.4 x 250 x 6 Cos α = 600 Cos α + 4500 Cos α

1500 Sin α = 4500 Cos α

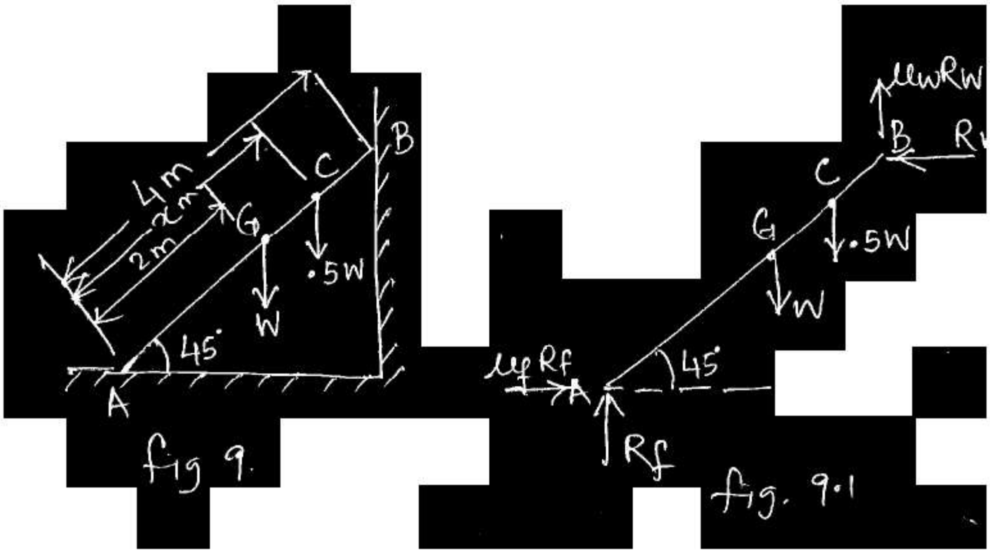
tan α = 3

α = 71.563^0

Ex – 7

A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45^0. The co-efficient of friction between the ladder and wall is 0.4 and that between ladder and the floor is 0.5. If a man, whose weight is half of that of the ladder ascends it, how height will it be when the ladder slips ?

Solution :



The FBD is shown in fig. 9.1

μ f = 0.5, μ w = 0.4

μ f Rf = Rw

.5 Rf = Rw -------------------(1)

Rf + μ wRw = w + 0.5 w = 1.5w

Rf + .5 x .4 Rf = 1.5 w

1.2 Rf = 1.5 w

Rf = 1.25 w ----------------------(2)

Rw = .5 x 1.25 w = 0.625w -------------------(3)

Taking moment about A

Rw x 4 sin 45 + μ w Rw 4 Cos 45^0

= w x 2 Cos 45 + .5 wx Cos 45^0

4 Rw + μ w4Rw = 2W + .5 xw

4 x .625 w + .4 x 4 0.625 w

= 2 w + .5 x w

2.5 + 1 = 2 + .5x

.5 x = 1.5

x = 3 m.

Wedge friction :

Wedges are generally triangular or trapezoidal in cross section. It is generally used for tightening keys or shaft. It is also used for lifting heavy weights. The weight of the wedge is very small compared to the weight lifted.

Let w = weight of the block

P = Force requried to lift the load

μ = co-efficient of friction on all contact surface.

α = Wedge angle.

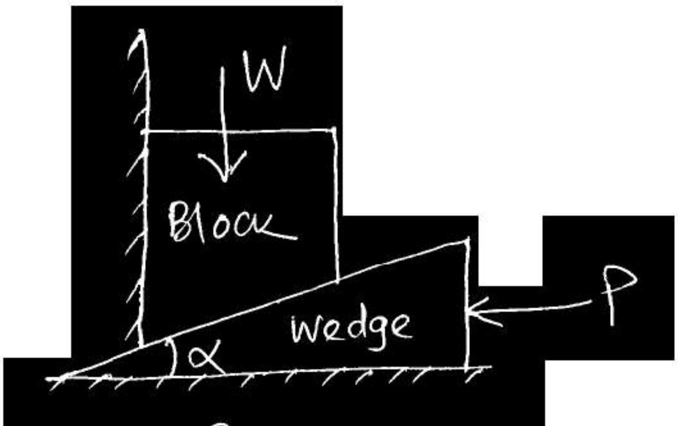
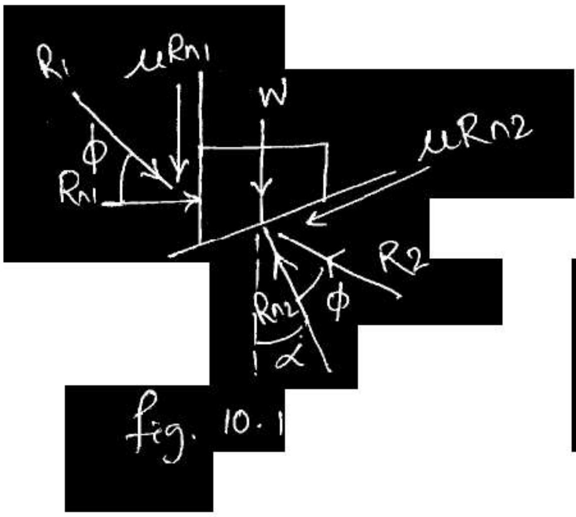
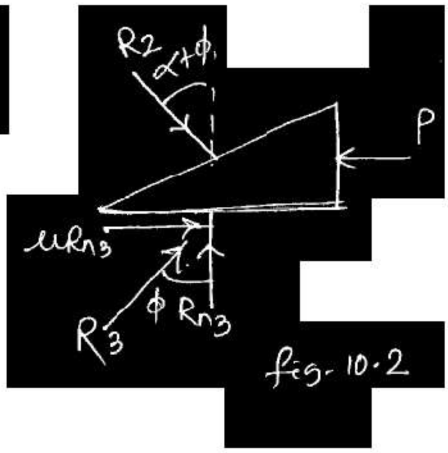


fig - 10



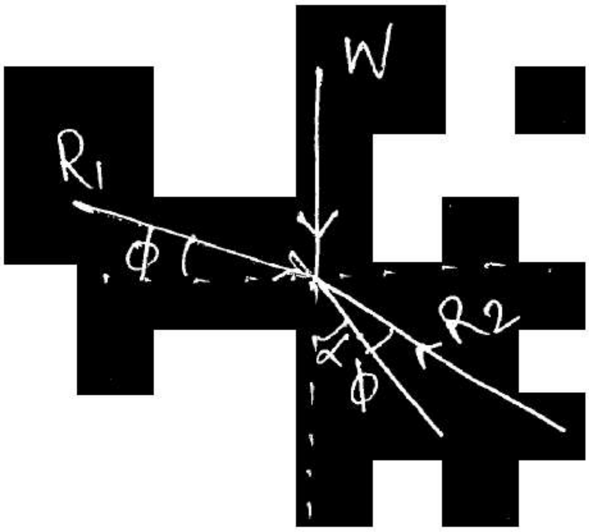


Considering the FBD of fig. 10.1 under the action of 3 forces the system is in equilibrium.

1) R_1 → Resultant friction of normal reaction and force of friction between block and wall.

2) W → Weight of the block.

3) R_2 → Resultant reaction of normal reaction and force of friction between contact surface of block and wedge.



Applying Lami's Theorem

$$\frac{W}{Sin[90+(\alpha+\phi)]}$$

$$= \frac{R_1}{Sin[180-(\alpha+\phi)]}$$

$$= \frac{R_2}{Sin(90-\phi)]}$$

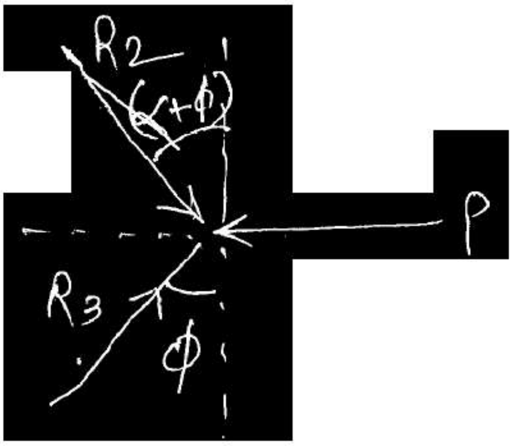
$$\text{or } \frac{W}{Cos(\alpha-2\phi)} = \frac{R_1}{Sin(\alpha+\phi)} = \frac{R}{Cos\phi}$$

Again considering the FBD of Fig. 10.2 under the action of 3 forces the system is in equilibrium.

1) R_2 → Reaction given by the block.

2) P → Force applied on wedge.

3) R_3 → Resultant reaction of normal reaction and force of friction between wedge and floor.



Applying lami's theorem

$$\frac{P}{Sin(\alpha+2\phi)}=\frac{R_2}{Cos\phi}=\frac{R_3}{Cos(\alpha+\phi)}$$