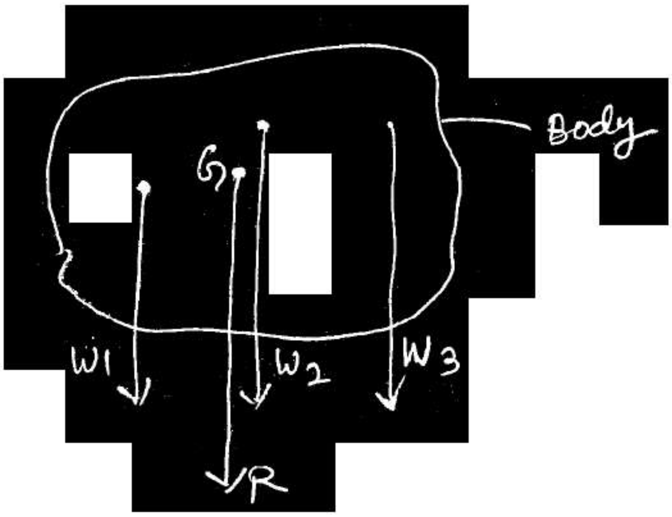
Chapter – 4

Centre d gravity – It is the point where whole of the mass of a body is supposed to act. In other words it is the point through which resultant of the parallel forces of attraction formed by the weight of the body, passes.

It is usually denotes by CG or simplify G.

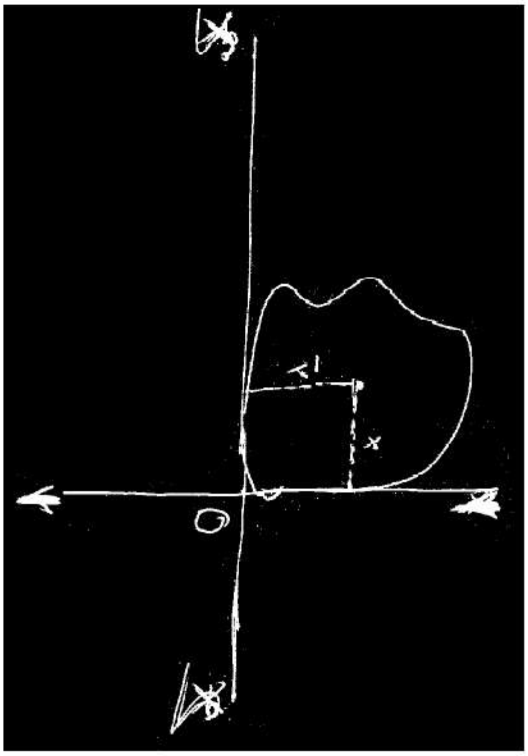


Centroid :

It is the point where the whole area of a body is assumed to be concentrated. Plane figures (known as laminas) have area only but no mass. The centre of gravity and centroid of such figures are the same point.

For plane areas, centroid is represented by two coordinates by selecting coordinate axes in the plane of the area itself.

The two co-ordinate axes are usually selected in the extreme left and bottom of the area.



Mathematically (continuous form)

$$\bar{x} = \frac{\int_A x.dA}{A}$$

Similarly $\bar{y} = \frac{\int_A y.dA}{A}$

$\int_A xdA$ and $\int_A ydA$ are known as first moment of area about x & y axes respectively.

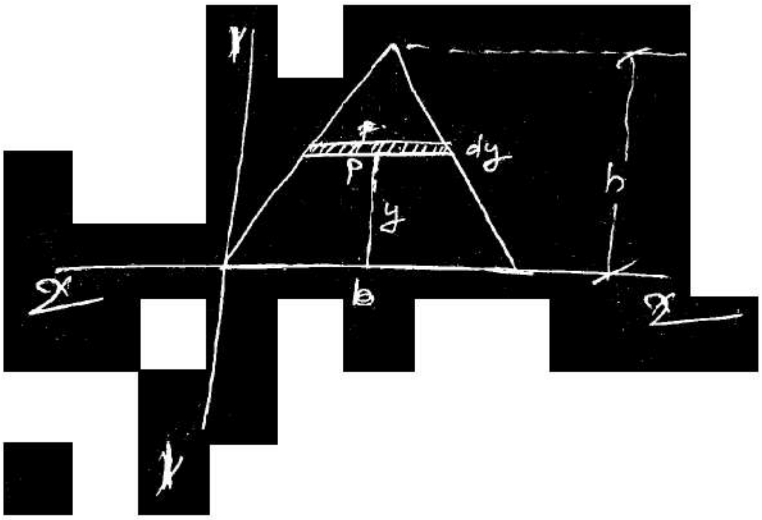
Mathematically – (Discrete form)

$$\bar{x} = \frac{\sum_{i=1}^{n} \Delta AiXi}{\sum_{i=1}^{n} \Delta Ai}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} \Delta AiYi}{\sum_{i=1}^{n} \Delta Ai}$$

Ex - 1

Determine the y - co-ordinate of the centroid of a uniform triangular lamina.



From the concept of similar triangles.

$$\frac{p}{b} = \frac{h-y}{h}$$

$$\therefore P = \left(\frac{h-y}{h}\right)b$$

$\therefore$ The area of the shaded portion dA = $\left(\frac{h-y}{h}\right)b.dy$

$$\therefore \ \bar{y} = \frac{\int_A y.dA}{A}$$

$$= \frac{\int_0^y y\left(\frac{h-y}{h}\right)b.dy}{\frac{1}{2}\times b.h}$$

$$= = \frac{\int_0^h y.b(1-y/h)b.dy}{\frac{bh}{2}}$$

$$= \frac{\cancel{b}\times\int y\left(1-\frac{y}{h}\right)dy}{\frac{\cancel{b}h}{2}}$$

$= \frac{2}{h}\left[ydy - \frac{y^2}{h}.dy \right]$

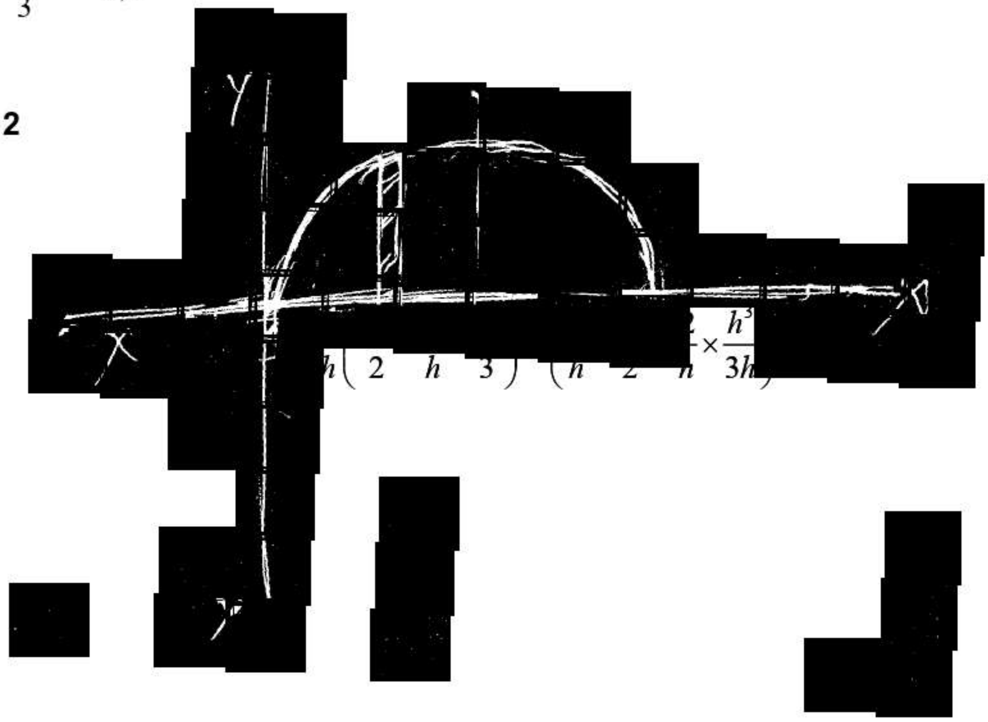
$= \frac{2}{h} \times \left[\frac{y^2}{2} - \frac{1}{h} \times \frac{y^3}{3} \right]_0^h$

$= \frac{2}{h}\left(\frac{h^2}{2} - \frac{1}{h} \times \frac{h^3}{3} \right) = \left(\frac{2}{h} \times \frac{h^2}{2} - \frac{2}{h} \times \frac{h^3}{3h} \right)$

$= h - \frac{2}{3}h$

$= \frac{3h - 2h}{3} = h/3$

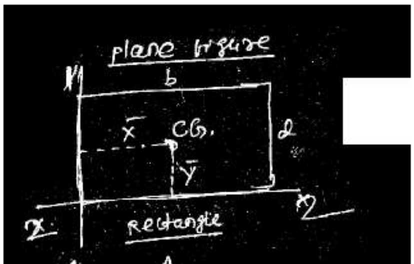
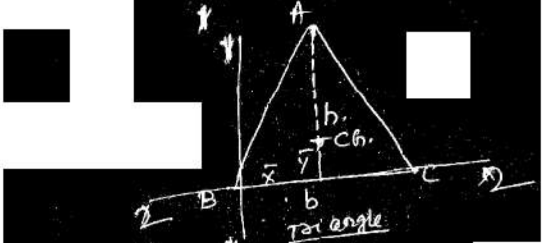
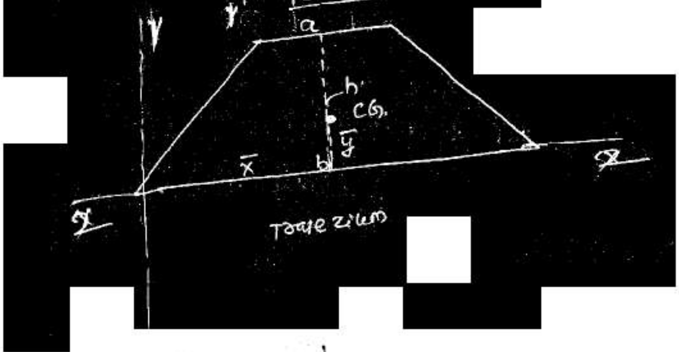
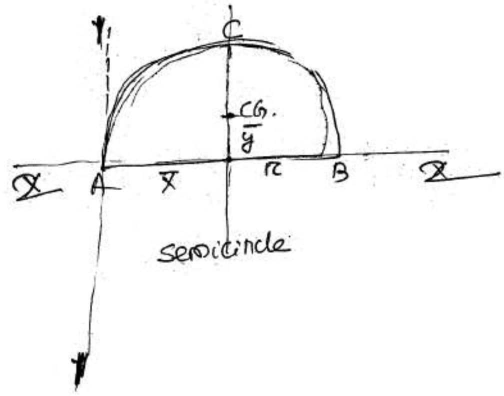
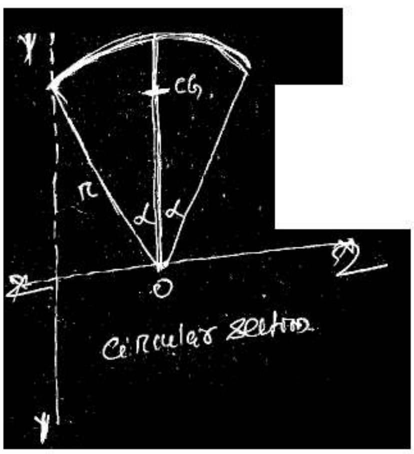
Ex- 2



$$\bar{x} = \frac{\int xdA}{\int dA}$$

$$\bar{y} = \frac{\int ydA}{\int dA}$$

A - Centre of gravity of some common figures :

Sl. No.	**Plane Figure**	**C.G. location**
1		$\bar{x} = \frac{b}{2}$ $\bar{y} = d/2$
2		$\bar{x} = b/2$ $\bar{y} = h/3\ (from\ \ base)$
3		$\bar{x} = b/2$ $\bar{y} = \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$
4		$\bar{x} = r$ $\bar{y} = \frac{4r}{3\pi}$
5		$\bar{y} = \frac{2}{3} r \frac{\sin\alpha}{\alpha}$

B - Centre of gravity of built up symmetrical sections.

Built up sections are formed by combining plane sections. The centre of gravity of such built up sections are calculated using method of moments.

The entire area of the builtup section is divided into elementary plane areas i.e. a_1, a_2, a_3 etc. Two reference axes are selected and let the distance of c.g. of these areas be x_1, x_2, x_3...... etc. respectively.

Let G be the centre of gravity of the entire area A and let the distances be $\bar{x}$ and $\bar{y}$ from y axis and x axis respectively.

Thus $A = a_1+a_2+a_3+.......$

Now sum of moment of all the individual areas about y-y- axis.

$$= a_1x_1+a_2x_2 + a_3x_3 +.....$$

and the moment of entire area about y y axis = $A\bar{x}$

Now equalising the two moment

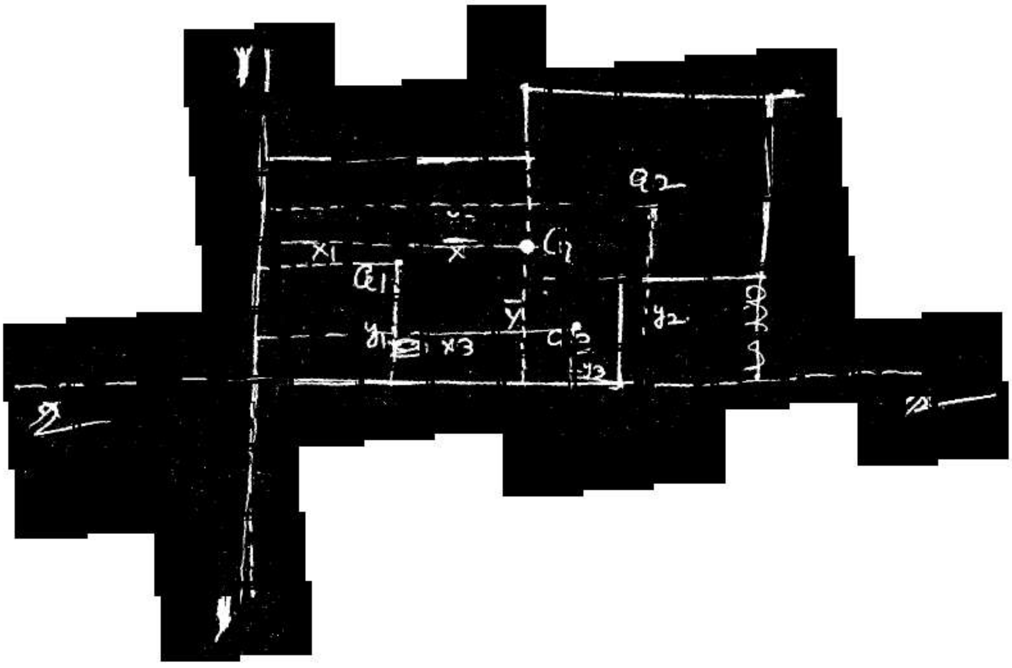
$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + ...$$

$$\therefore \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + ...}{A}$$

$$\frac{a_1x_1 + a_2x_2 + a_3x_3 + ...}{a_1 + a_2 + a_3}$$

Similarly

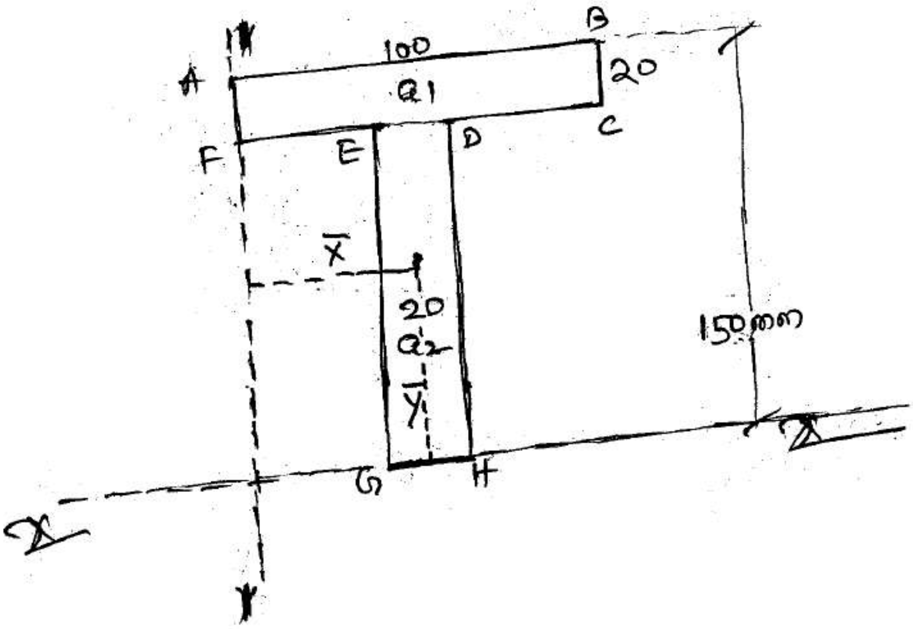
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + ...}{a_1 + a_2 + a_3}$$



Example :

Locate the centre of gravity of a T-Section having dimension

100mm x 150mm x 20mm



Two reference axes are selected i.e. y-y and x-x at the extreme left and bottom of the given section.

The entire area of the T- Section is divided into two portions.

i.e.

a_1= Area of the rectangle ABCD

= 100 x 20

= 2000 mm^2

y_1 = Distance of c.g. of the rectangle ABCD from x-x axes

= 140 mm

a_2 = Area of the rectangle EDHG

= 20 x 130

= 2600 mm^2

y_2 = distance of c.g. of the rectangle EDHG from x-x axes

= 65 mm

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{2000 \times 140 + 2600 \times 65}{2000 + 2600} = \frac{280000 + 169000}{4600}$$

= 97.61 mm

As the given T-section is symmetrical about y-y axis,

$\bar{x} = 50mm$

Centre of gravity of section with cutout

Many a times some portions of a section are removed and thus the centre of gravity of the remaining portion is shifted to new locations.

The c.g. of such a section with cutout can be found out by considering the entire section first and then deducting the cutout section.

If the area of the main section and cut out are a_1 and a_2 respectively and c.g. distances are x_1, y_1 and x_2, y_2 from the reference lines, then the C.G. of the out section from the reference lines.

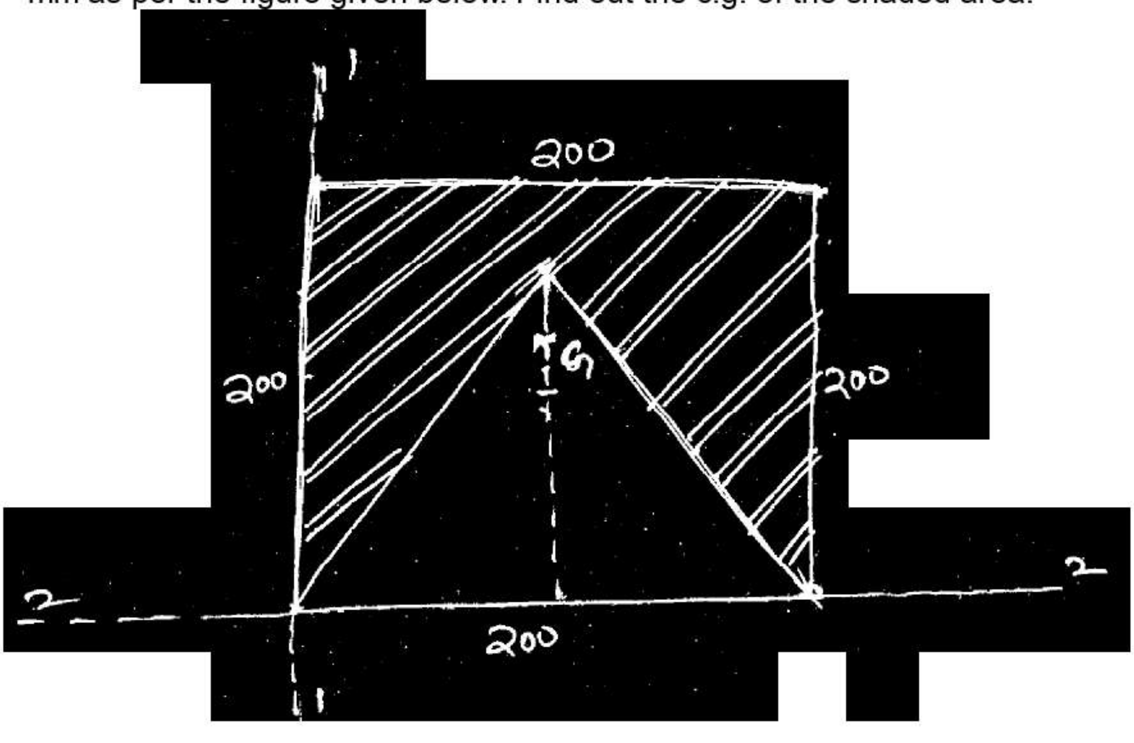
$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

&

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

Ex-

An isosceles triangle of side 200 mm has been cut from a square ABCD of side 200 mm as per the figure given below. Find out the c.g. of the shaded area.



Here,

$a_1 = 200 \times 200$

$= 40{,}000$ sqmm

$x_1 = 100$ mm

$y_1 = 100$ mm

$a_2 = 1/2 \times 200 \times 173.2$

$= 17{,}320$

$x_2 = 100$

$y_2 = 173.2 / 3$

$= 57.73$ mm.

$$\therefore \bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{40{,}000 \times 100 - 17320 \times 100}{40{,}000 - 17{,}320}$$

=100 mm

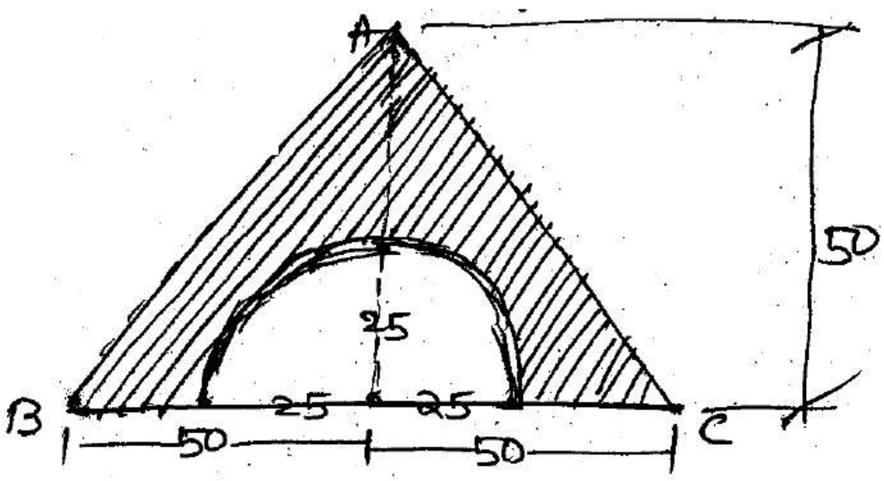
$$\bar{y} = \frac{40{,}000 \times 100 - 17{,}320 \times 57.73}{40000 - 17{,}320}$$

$$= \frac{4000000 - 999883.60}{22{,}680}$$

=132.28 mm

Ex

Locate the centrod of the cut out section (Shaded area) as shown in the figure.



Here

$$a_1 = \frac{1}{2} \times 100 \times 50$$

= 2500 sqmm

$$y_1 = \frac{h}{3}$$

$$= \frac{50}{3}$$

= 16.67 mm

$$a_2 = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times 25^2}{2}$$

$= 981.75\ mm^2$

$$y_2 = \frac{4r}{3\pi}$$

$$= \frac{4 \times 25}{3 \times \pi}$$

= 10.61 mm

$$\therefore y_1 = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

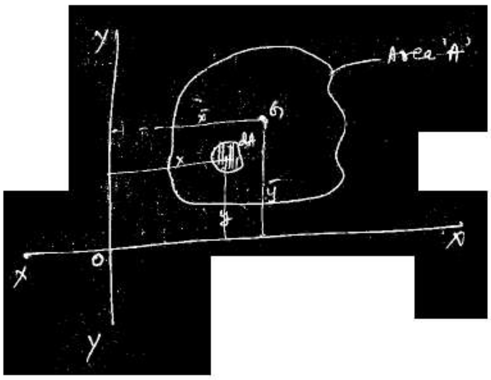
$$= \frac{2500 \times 16.67 - 981.75 \times 10.61}{2500 - 981.75}$$

$$= \frac{41675 - 10,416.37}{1518.25}$$

= 20.6 mm

Moment of inertia (Second moment) of an area :

Moment of interia of any plane area A is the second moment of all the small areas dA comprising the area A about any axis in the plane of ara A.



Referring to the figure :

I_{yy} = moment of interia about the axis y-y

$= \sum x^2 dA$

$= \int x^2 dA$

Similarly :

Ixx = Moment of interia about the axis x-x

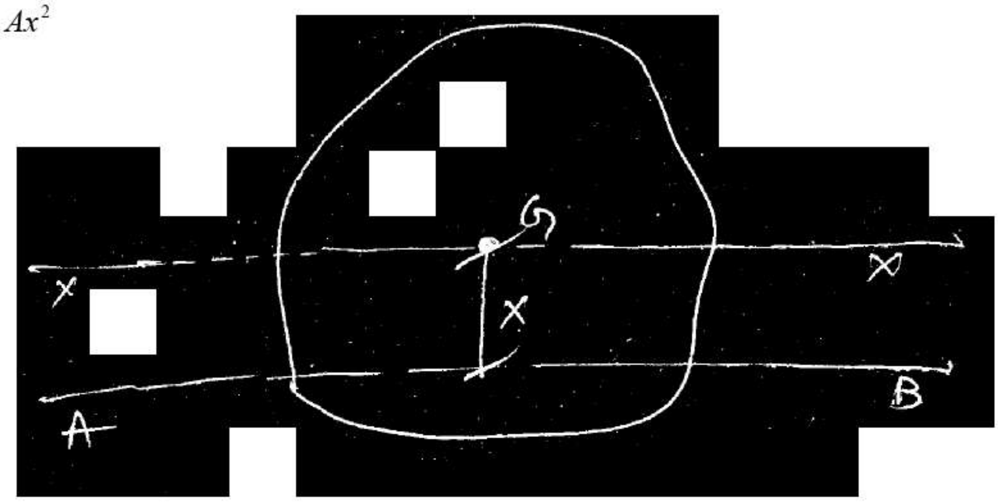
$= \sum y^2 dA$

$= \int_A y^2 dA$

Parallel axis theorem

If the moment of inertia of a plane area about an axis through the centroid is known, the moment of inertia about any other axis parallel to the given centroidal axis can be found out by parallel axis theorem. It states that if 1_{cg} is the moment of inertia about the centroidal axis, then moment of inertia about any other parallel axis say AB at a distance of x from the centroidal axis is given by.

$I_{AB} = I_{xx} + Ax^2$



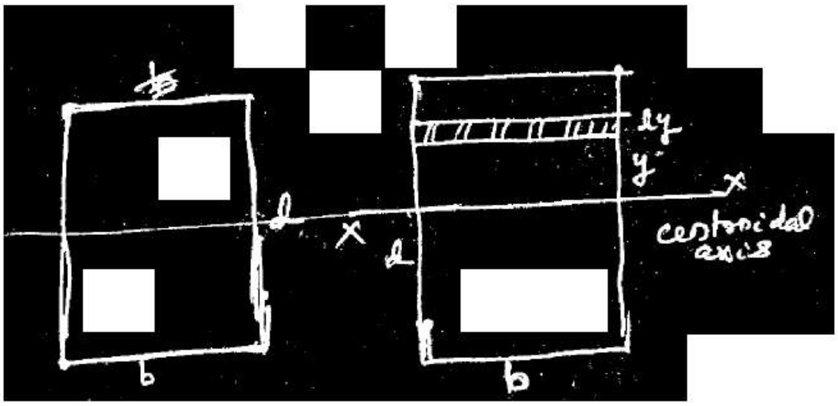
Perpendicular axis theorem

It states that second moment of an area about an axis perpendicular to the plane of the area through a point is equal to the sum of the second moment of areas about two mutually perpendicular axes through that point.

The second moment of area about an axis perpendicular to the plane of the area is known as polar moment of inertia.

$I_{zz} = I_{xx} + I_{yy}$

Moment of interia of a rectangular section :



Let us consider as elementary strip of thickness dy at a distance of y from the centroidal axis x-x.

$\therefore$ Area of the strip of this elementary area = b.dy

& MI about x x - axis = $A.y^2$

$= b.\ dy.\ y^2$

Now M.I. of the entire area.

$= I_{xx}$

$= \int_{-d/2}^{d/2} by^2 dy = b.\frac{y^3}{3} \int_{-d/2}^{d/2}$

$= \frac{bd^3}{12}$

Similarly it can be shown that $I_{yy} = \frac{db^3}{12}$

Let us consider a circle of radius r and an elementary ring of thickness dy a distance of y from the centre '0'.

Now area of this elemental ring = dA

$$= 2\pi y.dy$$

M.I. of this ring about '0' = $2\pi y.dy.y^2$

and MI of the entire area i.e. $Izz = \int_0^r 2\pi y.dy.y^2$

$$= 2\pi \times \int_0^r y^3 dy$$

$$= 2\pi \times \frac{r^4}{4}$$

$$= \frac{\pi r^4}{2}$$

$$\therefore Ixx = Iyy = \frac{I_{zz}}{2}$$

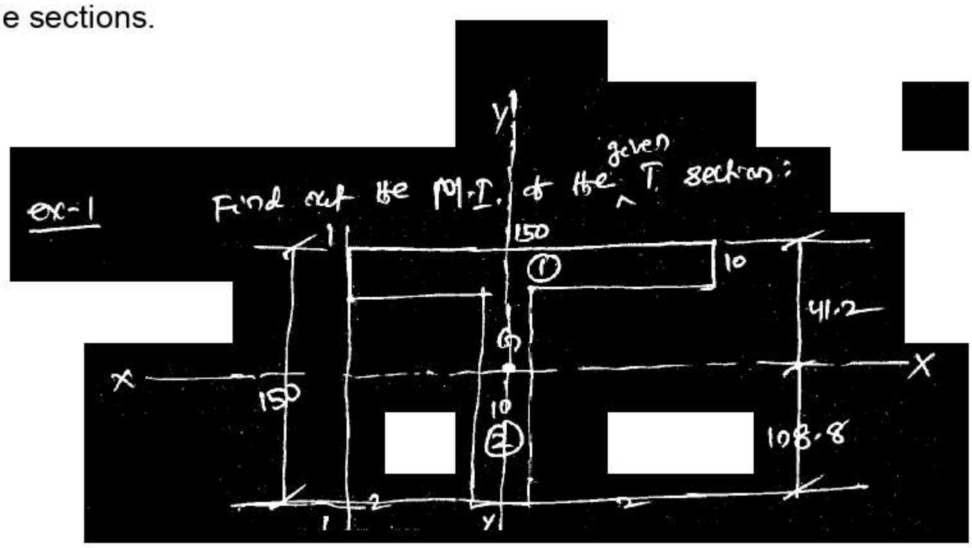
$$= \frac{\pi r^4}{2} \times \frac{1}{2}$$

$$= \frac{\pi r^4}{4}$$

$$= \frac{\pi d^4}{64}$$

Moment of inertia of built up section :

The second moment of area of a build up section which consists of a number of simple sections can be found out as the sum of the second moments of area of the simple sections.



$\bar{x} = 75mm$ (from the reference axis 1-1)

$\bar{y} = \dfrac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$ (from the reference axis 2-2)

$= \dfrac{150 \times 10 \times 145 + 140 \times 10 \times 70}{150 \times 10 + 140 \times 10}$

$= \dfrac{217500 + 98000}{1500 + 1400}$

= 108.8 mm

Moment of inertia of the T section can be found out by adding the M.I. of the two simple rectangular sections.

M.I. of the 1st portion about the axis x-x

$= \dfrac{150 \times 10^3}{12} + 150 \times 10 \times (41.2 - 5)^2$

= 12,500 + 19,65,660

= 19,78,160

M.I. of the 2nd portion about the axis x-x

$\dfrac{10 \times 140^3}{12} + 10 \times 40 \times (108.8 - 70)^2$

= 22,86,666.7 + 21,07,616

= 43,94,282.7 mm^4

$\therefore$ M.I. of the T section about xx - axis

$I_{xx} = 1978160 + 4394282.7$

= 63, 72, 442.7 mm^4