SIMPLE MACHINES (ENGG. MECHANICS)

Chapter 5

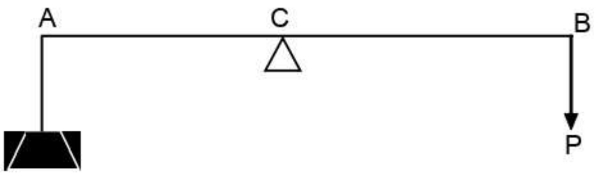
Before going to details of Simple Machines, we should know the following terms and their definitions.

5.1. Important Terms :

1. Simple Machine :

It is a contrivance in which an external force is applied at some point in order to overcome a bigger resistance at some other point.

Example



Taking moment about C, we get W x AC = P x BC

BC > AC

$$\frac{W}{P} = \frac{BC}{AC}$$

W>P or P<W

W - resistance or load

P - external force or effort

2. Lifting Machine :

It is a Simple machine in which the load lifted acts as a resistance.

3. Mechanical Advantage :

Briefly known as M.A. is the ratio of the weight lifted to the effort applied.

$$M.A = \frac{W}{P}$$

No unit, it is a pure number.

4. Input of a Machine

The input of a machine is the work done on the machine.

In a lifting machie, it is measured by the product of the effort and the distance through which it has moved.

Input = P x Y - **N.M.**

Whered - P = Effot in N and Y= distance moved by effort in M.

5. Output of a Machine

In a lifting machine, it is measure by the product of the weight lifted and the distance through which it has been lifted.

Input = W x X - **N.M.**

Where W is load in N and X is distance in M which the load is lifted.

6. Efficiencty of a Machine

It is the ratio of output to the input of a machine.

Mathematically, $\eta = \frac{\text{output}}{\text{Input}}$

$$\therefore \frac{W \times X}{P \times Y} = \frac{\left(\frac{W}{P}\right)}{\left(\frac{Y}{X}\right)} = \frac{M.A}{V.R}$$

7. Ideal Machine

If the efficiency is 100% i.e., out put = Input, the machine is called as a perfect or an ideal machine.

8. Velocity Ratio (V.R)

It is the ratio of the distance moved by the effort (y) to the distance moved by the load (x).

Mathematically, $V.R = \frac{Y}{X}$

9. Effort lost in Friction

Let P_1 = effort required to lift the same load W under ideal condition the Effort lost in Friction.

$P - P_1$

In ideal case, input = output

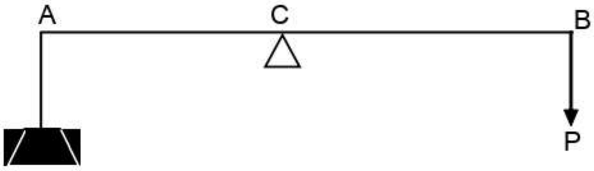
$P_1 \times Y = W \times X$

$P_1 \frac{WX}{Y} = \frac{W}{\left(\frac{Y}{X}\right)} = \frac{W}{V.R}$

Effort lost in Friction = $P - P_1$

$= P = \frac{W}{V.R}$

10. Reversibility of a lifting Machine



When the effort P is withdrawn the end A will go down wards raising the end B upwards. We say that the lever is reversible.

So a reversible machine is that machine in which the load moved in the reverse direction after withdral of the effort which was applied to raise the load.

Work lost in Friction = P x Y - Wx

In order that, the load may fall back after withdrawal of the effort.

So W.X > Py - Wx

or 2Wx > Py

or $\frac{Wx}{Py} \rangle \frac{1}{2}$

Efficiency > $\frac{1}{2}$ or Efficiency > 50%

Condition of Reversibility

The machine will be revresible when its efficiency is > 50%

11. Self locking Machine

If a Machine is not capable of doing any work in the reverse direction after the effort in withdrawn and such a machine is called a non-reversible or self locking machine.

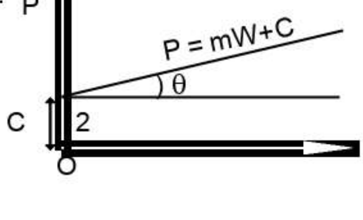
For this $\eta < 50\%$

12. Low of a lifting Machine

Where P= effort applied ,W= load lifted ,M and C are constants.

C- the effort required to move the machine under no load. i.e., to overcome friction.

13. Maximum Mechanical Advantage of a lifting Machine.



we know, $M.A = \frac{W}{P}$

Also we knwo, P = MW + C

Then putting it

$$M.A = \frac{W}{MW + C} = \frac{1}{M + \frac{C}{W}}$$

neglecting $\frac{C}{W}$

$$(M.A)_{max} = \frac{1}{M}$$

14. Maximum Efficiency of a lifting Machine

we know that

$$\eta = \frac{W/P}{VR} = \frac{W}{P \times VR}$$

Putting for max efficiency

$$\eta_{max} = \frac{W}{(MW + C)VR} = \frac{1}{\left(M + \frac{C}{W}\right)VR}$$

neglecting $\frac{C}{W}$, $\eta_{max} = \frac{1}{M \times VR}$

Example

In a lifting machine, an effort of 31 N raised a load of 1 KN. If efficiency of the machine is 0.75, what is its VR ? If on this machine, an effort of 61 N raisd a load of 2 KN, what is now the efficiency ? what will be the effort required to raise a load of 5 KN ?

Solution

Data given P_1 = 31 N, W_1 = 1 KN = 1000 N.

η = 0.75, P_2 = 61 N, W_2 = 2 KN = 2000 N.

η = ?, VR_1 = ?, P_3 = ?, W_3 = 5 KN = 5000 N.

VR

$$\eta_1 = \frac{MA}{VR} = \frac{\frac{1000}{31}}{VR}$$

$$\text{or } 0.75 = \frac{1000}{31 \times VR}$$

$$\text{or } VR = \frac{1000}{31 \times 0.75} = 43 \text{ (Ans)}$$

η_2

$$\eta_2 = \frac{MA}{VR} = \frac{2000}{61 \times 43} = 0.763 \text{ or } 76.3\% \text{ (Ans)}$$

P3 = ?

P = mw +c

31 = m x 1000 +c ...... (i)

61 = m x 2000 +c ......(ii)

substracting 9i) from (ii)

61-31 = 2000 m - 1000m

$\Rightarrow 30 = 1000m$

$$\Rightarrow m = \frac{3\theta}{1000} = 0.03$$

We have 31 = 1000 M + C

31 = 1000 x .03 +C

$\Rightarrow$ 31 =30 + C

$\Rightarrow$ C = 1

Then P = 0.03 W +1

Now for 5000N, P= 0.03 x 5000 +1

$\Rightarrow$ P = 151 N **(Ans)**

5.2. Study of Simple Machine

Types

1. Simple wheel and axle.
2. Differential wheel and axle.
3. Weston's differential pulley block.
4. General pulley block.
5. Worm and worm wheel.
6. Worm geared pulley block.
7. Single purchase crab winch.
8. Double purchase crab winch.
9. Pulleys :
   (a) First system of Pulleys
   (b) Second system of Pulleys
   (c) Third system of Pulleys
10. Simple Screw Jack
11. Differential Screw Jack
12. Worm geared Screw Jack.

1. Simple wheel and Axle

In Fig 2.1 is shown a simple wheen and axle, in which the wheel A and axle B are keyed to the same shaft. A string is wound round the axle B, which carries the load W to be lifted. A Second string is wound round the wheel A in the opposite direction to that of the string on B.

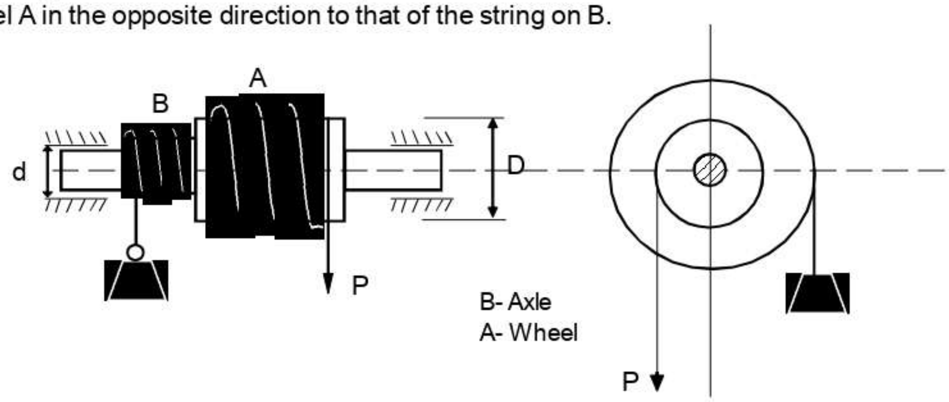


Fig 2.1 Simple wheel and axle

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since th two strings are wound in opposite directions, therefore, a downward motion of P will raise the load W.

Let D = Diameter of effort wheel

d = Diameter of the load axle

w = Load lifted, and

p = Effort applied to the load.

Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel makes one revolution the axle will also make one revolution.

In one revolution of effort wheel A, displacement of effort similarly the load displancement $= \pi D$

$$VR = \frac{\pi D}{\pi d} \times \frac{D}{d}$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{M.A}{VR}$$

If t_1 = thickness of the rope used over the wheel.

and t_2 = thickness of the rope used over the axle.

$$\text{then } VR = \frac{D + T_1}{d + T_2}$$

Example

In a wheel and axle machine, the diameter of the wheel is 100cm and that of the axle is 10 cm. The thickness of the load on the wheel is 3 mm and that on the drum is 6 mm. In this machine a load of 500 N is lifted as an effort of 100 N. Determine the efficiency and state whether the machine is reversible at this load.

Data given : D = 100 cm, W = 500N

d = 10 cm, P=100N

T_1 = 3mm

T_2= 6mm

To find, n = ?

$$MA = \frac{W}{P} = \frac{500}{100} = 5$$

Solution

$$MA = VR = \frac{D + t_1}{d + t_2}$$

$$\Rightarrow VR = \frac{(100 + 0.3)cm}{(10 + 0.6)cm} = 9.46$$

$$\text{and } \eta = \frac{MA}{VR} = \frac{5}{9.46} \times 100 = 52.85\% \text{ (Ans)}$$

Since efficiency is greater than 50%, the machine is reversible.

Double Purchase crab

Double or Purchase winch

Diagram

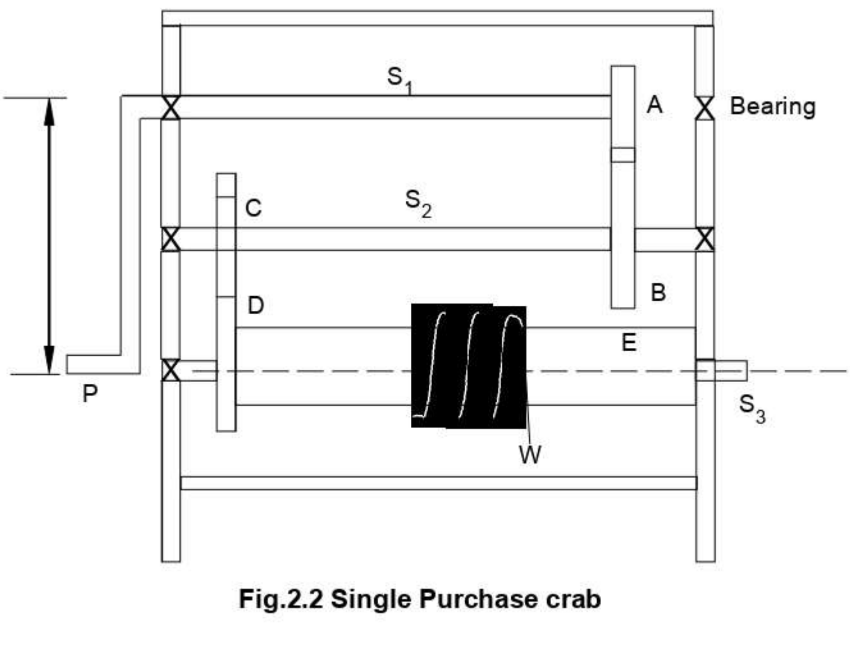


Fig.2.2 Single Purchase crab

Single purchase crab consists of a Pinion (A), which is engaged with a Spur gear B. Gear A and the effort handle are fixed to the same shaft(G_1). Also, adrum C and the gear b are fixed to the same shaft (S2). Load is lifted by a string wound round the drum.

Calculation

Let T_A = number of teeth of Pinion(A)

T_B = number of teeth of the gear (B)

L = Length of the effort handle

D = diameter of the load drum.

When the effort handle is rotated once, the distance moved by effort = $= 2\pi L$ when the handle is rotated once, the Pinion (A) also rotates once, i.e., N_A=1rpm, where N_A= revolution of Pinion A.

Let N_B=revolution of gear B

Then we know for gear Drive

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

$$\therefore N_A \times \frac{T_A}{T_B} = 1 \times \frac{T_A}{T_B} = \frac{T_A}{T_B}$$

Revolution of Pinion C is the same as that of gear (B) as both are connected to same shaft.

$$\text{So } N_C = \frac{T_A}{T_B}$$

Again we know,

$$\frac{N_D}{N_C} = \frac{T_C}{T_D}$$

$$\therefore N_D = N_C \times \frac{T_C}{T_D}$$

Putting N_C value

$$\Rightarrow N_D = \frac{T_A}{T_B} \times \frac{T_C}{T_D}$$

Super gear (D) and a drum (E) are mounted on a third shaft S_3.

So when the effort handle makes one revolution, the gear D and drum E make $\frac{T_A T_C}{T_B T_D}$ revolution.

So, the load is lifted through a distance $= \pi D \times \frac{T_A T_C}{T_B T_D}$

Where D= diameter of the load drum E

$$\text{Hence Velocity Ratio} = \frac{2\pi \ell}{\pi D \times \frac{T_A T_C}{T_B T_D}}$$

$$\boxed{VR = \frac{2\ell}{D} \times \frac{T_B T_D}{T_A T_C}}$$

Example

In a double purchase crab, the pinions have 15 and 20 teeth while the spur wheels have 45 and 42 teeth. The effort handle is 40 cm. While the effective diameter of the drum is 15 cm, if the efficiency of the winch is 40%, what load will be lifted by an effort of 250N applied at the end of the handle ?

Data Given

TA = 15, TB = 45

TC =20, TD = 40

L=40 cm D = 15 cm

η=40%, P=250 N

To find W =?

Solution

we know,

$$VR = \frac{2\ell}{0} \times \frac{T_B T_D}{T_A T_C}$$

$$\text{Or } VR = \frac{2 \times 40}{15} \times \frac{45 \times 40}{15 \times 20} = 32$$

$$\eta = \frac{M.A}{VR} = \frac{W/250}{15}$$

$$\text{Or } 0.4 = \frac{W}{250 \times 32}$$

$$\Rightarrow W = 3200N = 32KN \text{ (Ans)}$$

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SINGLE PURCHASE CRAB WINCH

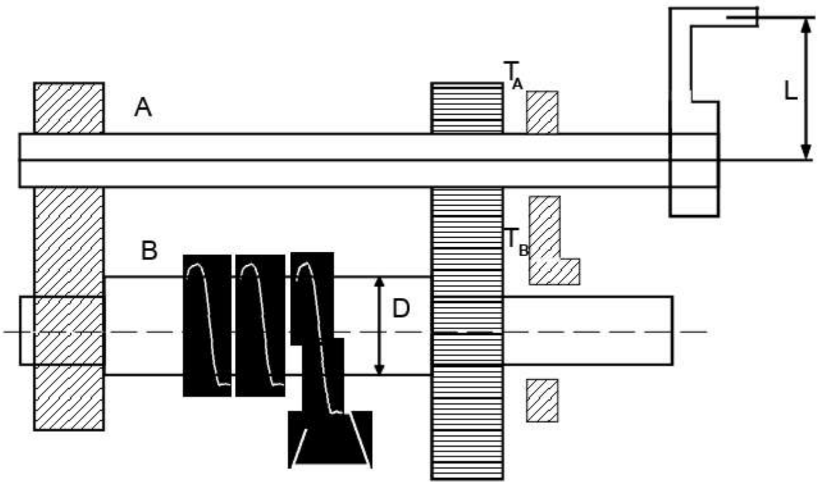


Fig 2:3 Single Purchase Crab Winch. In a Single Purchase Crab Winch, a rope is fixed to the drum and wound. A spur gear (B) is mounted on the load drum. Another pinion A is geared with (B) and connect to effor wheel.

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

If, $N_A = 1$

$$N_B = \frac{T_A}{T_B}$$

Distance moved by load in N_B revolution

$$= \pi D \times \frac{T_A}{T_B}$$

$$VR = \frac{Y}{X} = \frac{2\pi\ell}{\pi D \times \frac{T_A}{T_B}} = \frac{2\ell}{D} \times \frac{T_A}{T_B}$$

<u>Example</u>

In a Single Purchase Crab, the number of Teeth on Pinion is 25 and on the Spur wheel 250, Radio of the drum and handle are 1500 mm and 300 mm respectively. Find the efficiency of the machine and the effect of friction, if an effor of 20 N can lift a load of 300 N.

Data given

$T_A = 25$

$T_B = 250$

D = 2 x 150 = 300 mm =30 cm

ℓ = 300 mm = 30 cm

To Find, η = ? and Friction = ?

Solution

$$VR = \frac{2\ell}{D} \times \frac{T_B}{T_A}$$

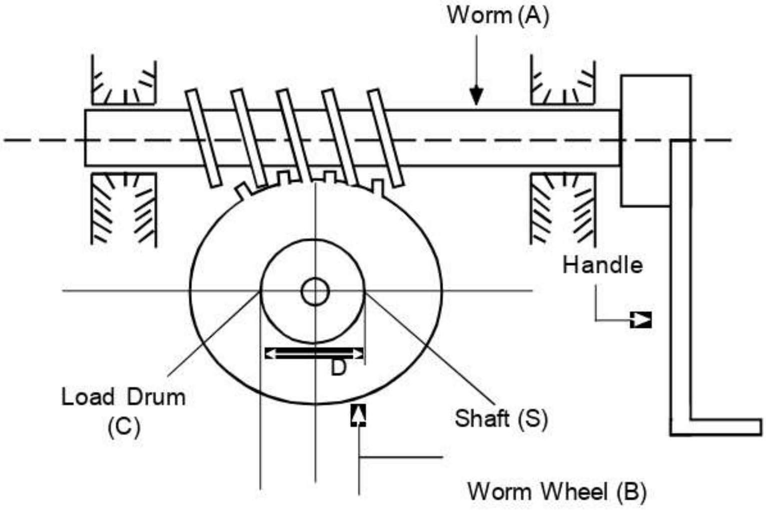
$$= \frac{2\times30}{30} \times \frac{250}{25} = 20$$

$$\eta = \frac{MA}{VR} = \frac{300/20}{20} = 75 = 75\% \text{ (Ans)}$$

Effort lost in Friction = $P - \frac{W}{VR}$

$$= 20 - \frac{300}{20} = 5N \text{ (Ans)}$$

Worm and Worm Wheel



Description

Worm is threaded spindle (A)

Worm wheel is a spur gear (B)

The threads of the worm are engaged within the teeth of the worm wheel. A load drum (C) is mounted on the same shaft(S) as that of the worm wheel. A string hangs vertically from the load drum string hangs vertically from the load drum (C) and the load (W) to be lifted is attached to the string at its free and as shown above. The effort (P) is applied at the end of a handle fitted at one end of the worm.

When the worm is rotated by application of effort at the end of the handle, the worm wheel and the drum (C) rotate winding the string round the surface of the drum. In this way the load is lifted the Worm(A) may have single start threads as 'multi start threads'.

If the worm has single start threads, in one rotation of the worm, one teeth.

If the worm has duble start threads is one rotation of the worm, two teeth of the worm wheel will move, and so on.

Culculation of VR

Let ℓ = Length of the effort handle

n = no, of starts of the worm

n = 1, single start thread

n = 2, double start thread

T = number of teeth of the worm wheel.

D = Diameter of the load drum (C)

Considering on rotation of the effort handle, the distance through which effort (P) moves $= 2\pi L$

We Know

$$\frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

$$\text{If } N_A = 1 \text{ rpm}$$

$$\Rightarrow N_B = 1 \times \frac{T_A}{T_B} = \frac{T_A}{T_B}$$

$$\text{Here for single start } T_A = 1$$

$$\therefore N_B = \frac{1}{T_B} = \frac{1}{T}$$

In one rotation of the load drum the load in lifted to a leight πD

In N_B rotation the load lifted will be $N_B \times \pi D = \pi D \times \frac{1}{T}$

In Multi start thread the load lifted will be = $\frac{\pi D \times n}{T}$

$$VR = \frac{2\pi L}{\pi D \times \frac{n}{T}} = \frac{2L}{D} \times \frac{T}{n}$$

$$\therefore VR = \frac{2LT}{nD}$$

Example

In a worm and worm wheel, the worm in triple threaded and the drum which is rigidly fixed to the wheel having common axis of rotation, has adiameter of 50 cm. Determineter the number of teeth on wheel of 40 turns of the worm move the load up by 60 cm. If the handle attached to the worm has an effective least of 30 cm and the load lifted weighs 25 KN. Calculate the M.A. and take efficiency as 40 %.

Solution

Given n = 3, D= 50cm, L=30 cm, W= 25 KN, n = 40 %

In 40 turns of the worm, distance described by effort = 40 x 2 π x 30 = 7539.83 cm

$$\text{Hence } VR = \frac{7539.82}{60} = 125.66$$

$$\text{But } VR = \frac{2LT}{Dn} = \frac{2 \times 30}{50} \times \frac{T}{3}$$

$$\Rightarrow T = 314.15$$

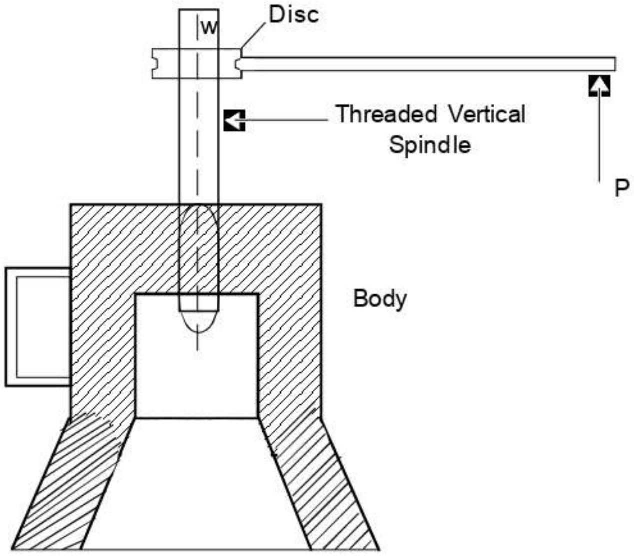
As teeth can not be fraction, T = 314.

$$\frac{MA}{VR} = \eta$$

$$\frac{MA}{125.66} = 0.4$$

$$\Rightarrow MA = 50.264 \text{ (Ans)}$$

SIMPLE SCREW JACK



A Simple Screw Jack consists a vertical threaded spindle. The treaded portion of the spindle passes throught a nut cut in the body of the Screw Jack.

The Load W is placed on a disc fitted at the top of the vertical spindle. The disc is fitted with a handle. Effort P is applied at the end of this handle. When the screw is truned by P, the screw either moves up or comes down throught a nut, depending upon the direction of rotation of the handle.

A Screw Jack is used to raise and hold up heavy machinary like motor car, truck, etc.

Calculation

Let ℓ = Length of the leaver

P = Pitch of the Screw

Velocity Ratio = $\frac{2\pi\ell}{P}$

Relation between Mean diameter Pitch and Heli x angle of a Screw thread

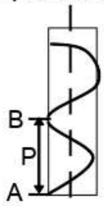
$D_m = \frac{D+d}{2}$

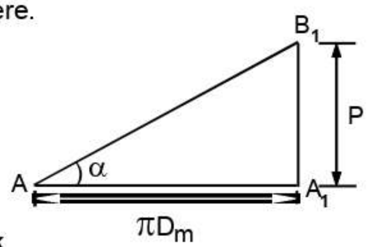
Where D- outside diameter

d - Inside diameter

Threads of a Screw may be considered to be a metal strip wrapped round a cylinder in the form of helix. The development of this helix in one pitch length is shown here.

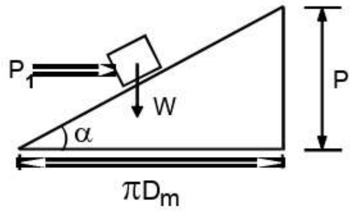
So $\tan\alpha \frac{P}{\pi D_m}$





Effort Required to Lift a load by means of a Screw Jack

When a load is lifted by means of a Screw Jack the case becomes equivalent to lifting a load (W) up an inclined plane of inclination α by applying a horizontal force. The equivalent inclined plane.



Let P_1 = horizontal roce required to move a body of weight W up the inclined plane.

Then $P_1 = W\ \tan(\alpha+\theta)$

Where θ = Angle of Friction.

In case of Screw Jack, P_1, acts at a distance of mean radins from the axis of the screw.

$$P\times\ell = P1\times\frac{D_M}{2}$$

$$= W\tan(\alpha+\theta)\times\frac{D_M}{2}$$

$$= W\times\frac{D_M}{2}\left(\frac{\tan\alpha+\tan\theta}{1-\tan\alpha\times\tan\theta}\right)$$

$$= W\times\frac{D_M}{2}\left(\frac{\frac{P}{\pi D_M}+\mu}{1-\frac{P}{\pi D_M}\times\mu}\right)$$

$$= W\times\frac{D_M}{2}\left(\frac{P+M\pi D_M}{\pi D_M\times\mu P}\right)$$

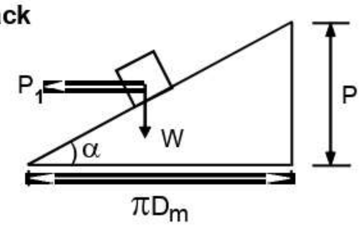
$$\text{or } P = W\times\frac{D_M}{2\ell}\times\frac{M\pi D_M+P}{\pi D_M\times\mu P}$$

Effort Required to Lower a load by Screw Jack

The Equivalent plane is shown below :

$$\text{Here} P\times\ell = P_1\frac{D_M}{2} = W\tan(\theta-\alpha)\times\frac{D_M}{2}$$

$$\text{or} P = W\times\frac{D_M}{2\ell} = \frac{\mu\pi D_M - P}{HD_M - \mu P}$$



Efficiencty when load is lifted

$$\eta = \frac{\text{output work}}{\text{Input work}} = \frac{W\times P}{P\times 2\pi\ell}$$

$$\text{Again } P\times\ell = P_1\frac{D_M}{2}$$