CHAPTER-5

ORTHOGRAPHIC PROJECTION

PROJECTION OF STRAIGHT LINES

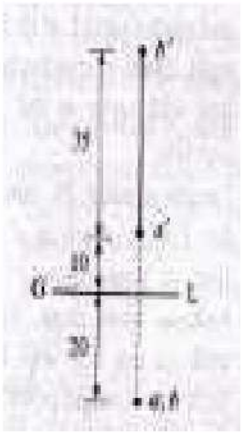
Lines in space are three types considering their position with respect to reference planes i.e. Horizontal Plane (HP) and Vertical Plane (VP):

1. Line parallel to one and perpendicular to other reference plane
2. Line parallel to both reference plane
3. Line parallel to one and inclined to other reference plane
4. Line inclined to both reference planes

Case-1 Line parallel to one and perpendicular to other reference plane

Example: 1

Draw the projection of a straight line AB of length 35mm, which is perpendicular to HP and parallel to VP. Point A is 10mm above HP and 20mm in front of VP.



Steps:

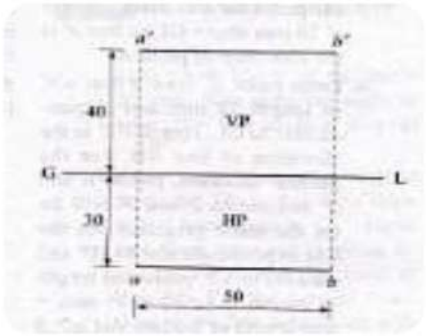
a. Draw the horizontal XY to represent reference line.
b. Locate a point 20mm below XY and mark as a. As the line is perpendicular to HP looking from the top both ends of the line will be projected as a single point. Hence the point b will coincide with a.
c. Draw a vertical projector through a and mark a point a' at 10 mm above the reference line to mark elevation of A.
d. Draw a line a'b' of length 35mm on the vertical projector and mark b' as the elevation of point B
   As the line is parallel to VP projected length in elevation (a'b') = 35mm= True length of the line.

*In case of line perpendicular to VP and parallel to HP the elevation of line a'b' will be a single line and the top view will be equal to true length of line drawn through the vertical projector through a' and drawn below reference line.

Case-2 Line parallel to both reference planes

Example: 2

Draw the projection of a straight line AB of length 50mm, which is parallel to both HP and VP. Point A is 40mm



above HP and 30mm in front of VP.

Steps:

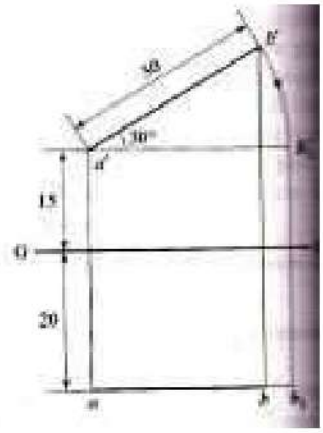
a. Draw the horizontal reference line XY.
b. Locate a point a 30mm below XY to represent top view of point A. Draw a line ab through a of 50mm long parallel to XY. Hence ab is the plan or top view of the line AB.
c. Draw a vertical projector through a and mark a point a' at 40mm above XY. Draw a line a'b' through a' of 50mm long parallel to XY. Hence a'b' is the elevation or front view of the line AB.
d. As the line is parallel to both HP and VP the projected length in top view (ab) =projected length in front view (a'b') = 50mm=true length of the line.

Case-3: Line parallel to one and inclined to other reference plane

Since the line is parallel to one reference plane the projection length on that plane will be equal to the true length of line and that projection will be drawn inclined to other plane. The projection length on the plane to which it is inclined will be of shorter length.

Example: 3

Draw the projection of a straight line AB of length 30mm, which is parallel to VP and inclined at 30^0 to HP. Point A is 15mm above HP and 20mm in front of VP.

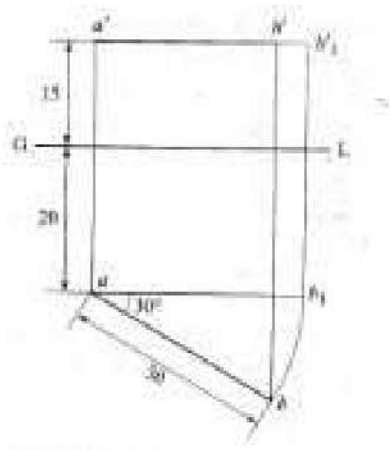


Steps:

a. Draw the horizontal reference line XY.
b. Locate a point a' 15mm above XY to represent front view of point A.
c. Through point a' draw a vertical projector and mark point a 20 mm below XY to represent top view of A.
d. Draw arbitrary horizontal line through a' parallel to XY. Through a' draw a line a'b' inclined at 30^0 with horizontal line such that a'b' = 30mm =True length of line, as the line is parallel to VP. The angle of elevation is = angle of the line with HP(Θ)=30^0
e. From point b' draw the vertical projector and through a draw a horizontal line parallel to XY. These two lines intersect at point b. hence ab is the top view of the line AB. As the line is inclined to HP the projected length is < true length of the line.

Example: 4

Draw the projection of a straight line AB of length 30mm, which is parallel to HP and inclined at 30^0 to VP. Point A is 15mm above HP and 20mm in front of VP.



Steps:

a. Draw the horizontal reference line XY.
b. Locate a point a' 15mm above XY to represent front view of point A.
c. Through point a' draw a vertical projector and mark point a 20 mm below XY to represent top view of A.
d. Draw arbitrary horizontal line through a parallel to XY. Through a draw a line ab inclined at 30^0 with horizontal line such that ab = 30mm =True length of line, as the line is parallel to HP. The angle of elevation is = angle of the line with VP(Ø)=30^0
e. From point b draw the vertical projector and through a' draw a horizontal line parallel to XY. These two lines intersect at point b'. Hence a'b' is the front view of the line AB. As the line is inclined to VP the projected length (front view) is < true length of the line.

Case-4: Line inclined to both reference planes

The projection of a straight line AB inclined to both HP and VP can be drawn with the following steps;

1. Assuming the line inclined to HP and parallel to VP draw its apparent front view $a'b_1'$ taking its true length and true angle of inclination (Θ) with HP. Draw the locus of point B in VP by drawing a horizontal line through b_1'.

2. Assuming the line inclined to VP and parallel to HP draw its apparent top view ab_2 taking its true length and true angle of inclination (Ø) with VP. Draw the locus of point B in HP by drawing a horizontal line through b_2.

To draw the length of the Top View

1. Draw a vertical projector through b_1' and it meets the horizontal line through point a at b_1. Taking a as centre ab_1 as radius draw the arc to meet the horizontal line through b_2 at b. The line ab is the top view of the line AB.

To draw the Front View of the line

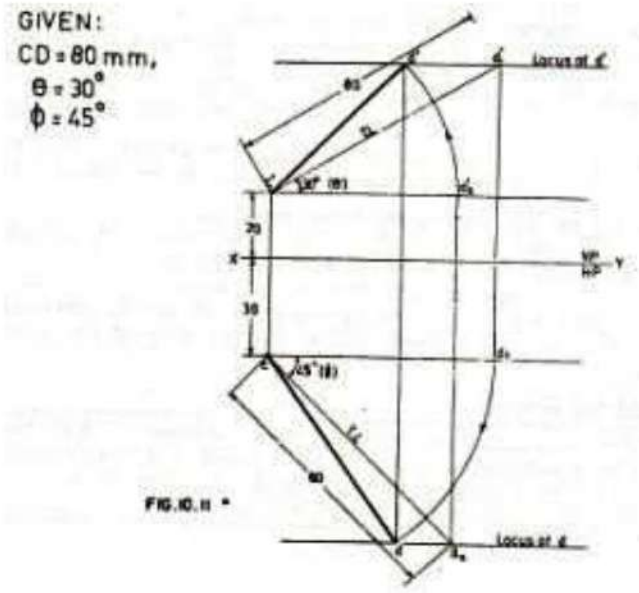
2. Draw a vertical projector through b_2 and it meets the horizontal line through point a' at b_2'. Taking a' as centre ab_2' as radius draw the arc to meet the horizontal line through b_1' at b'. The line a'b' is the front view of the line AB.

3. The vertical projector aa' and bb' are end projectors of point A and B respectively.

4. The horizontal distance between aa' and bb' is the distance between end projectors of A & B.

Example: 5

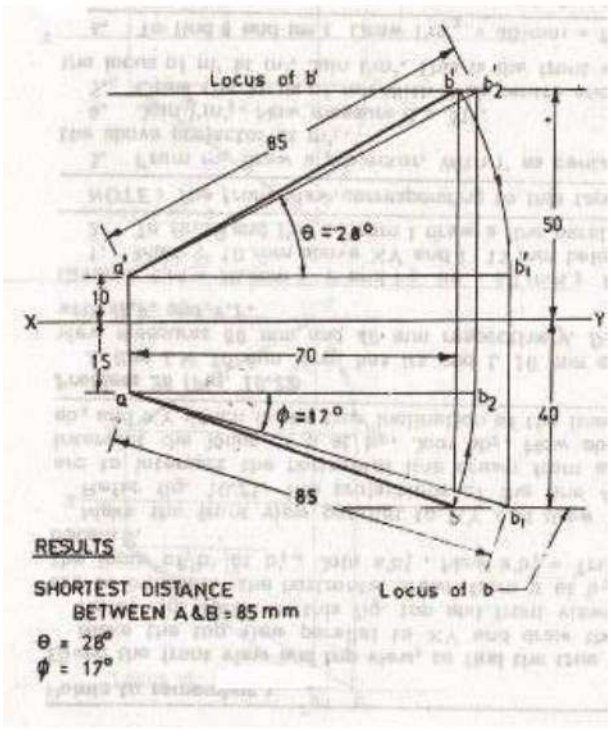
A line CD 80mm long is inclined at 30^0 to HP and 45^0 VP. The point C is 20mm above HP and 30mm in front of VP. Draw the projections of the line.



a. Draw the horizontal reference line XY.
b. Mark c' 20mm above XY.
c. On the vertical projector through a', mark c 30mm below XY. From c' draw a horizontal line and draw a line at 30^0 to XY and mark d_1' such that $c'd_1'$= True length=80mm. Draw the locus of point D in VP by drawing a horizontal line through d_1'.
d. From c draw a horizontal line and draw a line at 45^0 to XY and mark d_2 such that cd_2= True length=80mm. Draw the locus of point D in HP by drawing a horizontal line through d_2.
e. From d_1' draw a projector to intersect the horizontal line through c at d_1. Now cd_1 is the length of top view. Taking c as centre and cd_1 as radius draw the arc to meet the horizontal line through d_2 i.e. locus of D in HP at d, such that cd is the top view.
f. From d_2 draw a projector to intersect the horizontal line through c' at d_2'. Now $c'd_2'$ is the length of front view. Taking c' as centre and $c'd_2'$ as radius draw the arc to meet the horizontal line through d_1'i.e. locus of D in VP at d', such that c'd' is the top view.
g. Now d and d' are on the same projector.

Example: 6

A line AB is 85 mm long. It's one end A is 10 mm above HP and 15mm in front of VP, while the other end B is 40 mm in front of VP and 50 mm above HP. Draw the projection of the line.

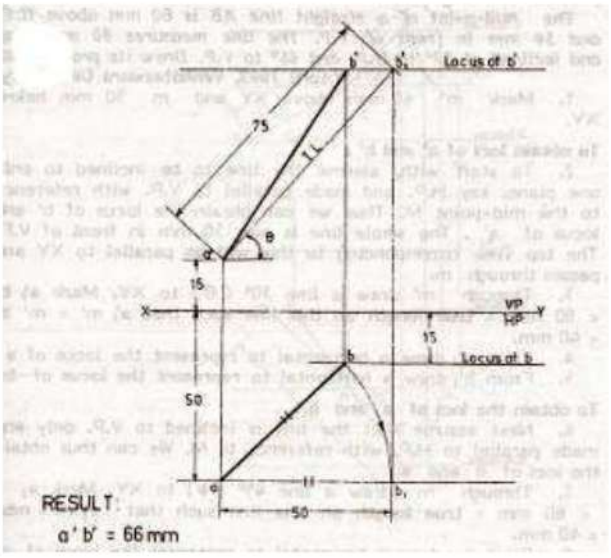


Steps:

1. Draw a horizontal line XY as the reference line.
2. Locate a point 10 mm above line XY and mark as a'.
3. Mark point a 15 mm below XY on the vertical projector through a'.
4. Draw an arbitrary horizontal line parallel to XY, which is 50 mm above which represents locus of b' in VP
5. Draw an arbitrary horizontal line parallel to XY, which is 40 mm below it which represents locus of b in HP.
6. Considering point a' as center and radius = 80 mm, draw an arc that intersects line locus of b' at point b_2'. Here a'b2' is the true length in elevation.
7. Considering point a as center and radius = 80 mm, draw an arc that intersects line locus of b at point b_1. Here ab_1 is the true length in plan.
8. Draw a horizontal line through a', from b_1 draw a perpendicular upward such that these intersects at b_1'.
9. Considering point a' as center and radius = a'b_1' draw an arc to touch line locus of b' at b'.
10. Hence a'b' is the front elevation.
11. Draw a horizontal line through a and from b_2' drop a perpendicular below such that these intersect at b_2.
12. Considering point a as center and radius = ab_2, draw an arc to touch line locus of b at b_2.
13. Hence ab is the top view.

Example: 7

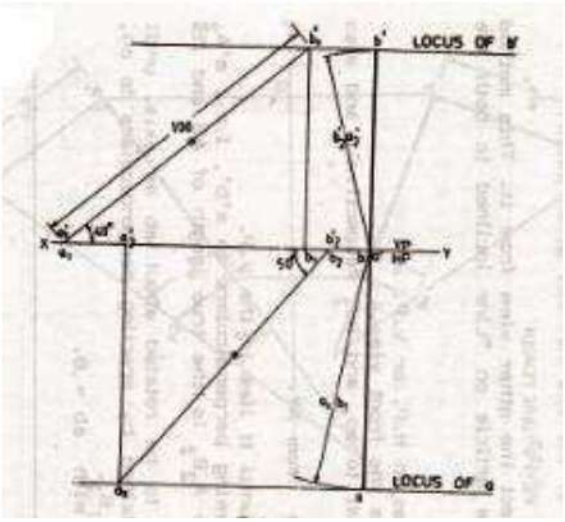
A line of length 75mm has one of its ends 50mm in front of VP and 15mm above HP. The top view of the line is 50mm long. Draw and measure the front view. The other end is 15mm in front of VP and is above HP.



1. Point A is 15mm above HP and 50mm in front of VP, mark a' 15 mm above XY and a 50 mm below XY on the same projector.
2. Point B is 15 mm in front of VP draw a horizontal line 15 mm below XY to represent locus of b.
3. The top view of the line is 50 mm. Taking a as centre and 50 mm as radius draw an arc to intersect the locus of b at b. Now ab is the top view of the line.
4. With a as centre and ab as radius draw an arc to intersect the horizontal line through a at b_1.
5. Taking a' as centre 75 mm as radius draw an arc to intersect the projector drawn from b_1 at b_1'.
6. Draw a horizontal line through b_1' to represent the locus of b'.
7. From b draw a projector to intersect the locus of b' at b'.
8. Join a'b' which is the front view of the line.

Example: 8

A line AB of length 100mm has one of its ends VP and the other end touching HP. The angle of inclination with HP and VP are 40^0 and 50^0 respectively. Draw the projections.

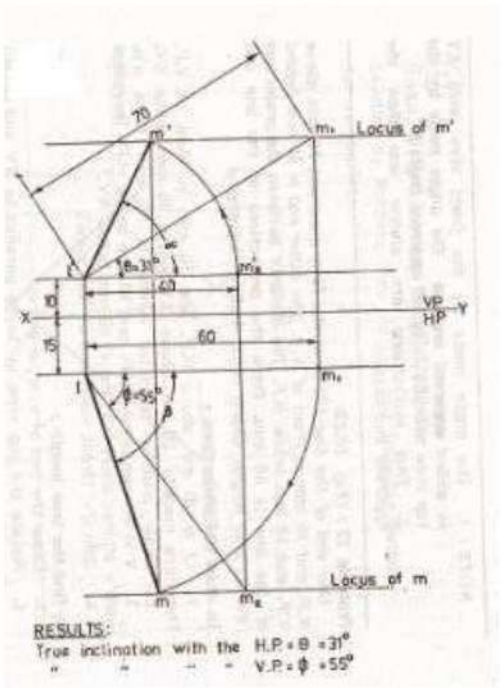


Steps;

1. Let A be the point touching the HP and B be the point touching VP. Thus a' and b will lie on XY.
2. With reference to B, rotate the line to be parallel to VP keeping $\Theta=40^0$.
3. Mark any point a_1' on XY to represent front view of A in above position. Draw the front view $a_1'b_1'$= 100mm=the true length and true inclination 40^0 with HP.
4. From b_1' draw a horizontal line to represent locus of b'.
5. a_1b_1 is the corresponding top view.
6. Take another point b_2 on XY. Draw a line from b_2 at 50^0(Ø) to XY and mark a_2 such that a_2b_2= 100mm=true length.
7. Draw a horizontal line through a_2 to represent locus of a.
8. Draw the front view $a_2'b_2'$ on XY corresponding to a_2b_2.
9. Mark any point a' on XY. With a' as centre and $a_2'b_2'$ as radius draw an arc to cut the locus of b' at b' such that $a'b'=b_1b_1'$. A'b' is the front view perpendicular to XY.
10. With b as centre and a_1b_1 as radius draw an arc to cut the locus of a at a,so ab is the to view, such that ab = $a_2'b_2'$

Example: 9

A line LM of length 70mm has its end L 10mm above HP and 15mm in front of VP. The top view and front view of the line measures 60mm and 40mm respectively. Draw the projections of the line and determine its inclinations with reference planes.



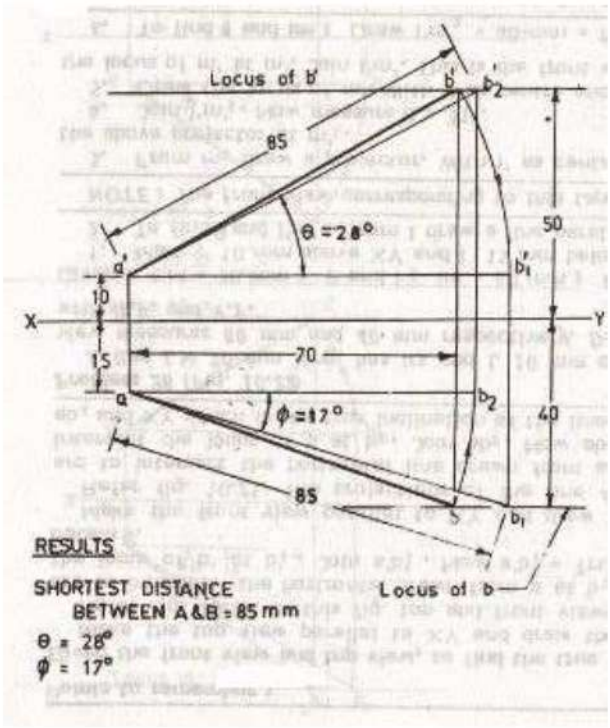
Steps;

1. Point L is 10mm above HP and 15mm in front of VP, mark l' 10 mm above XY and l 15 mm below XY on the same projector.
2. From l draw a line parallel to XY and mark lm_1=60mm=top view length

3. From m_1 draw a projector. With l' as centre and true length =70mm as radius draw an arc to intersect the above projector at m'_1.
4. Join l' m'_1 and measure 'Θ'.
5. Draw the locus of m' through m'_1. With l' as centre and front view length=40mm as radius draw an arc to intersect the locus of m' at m'. Join l'm', which is the front view.
6. Draw $l'm'_2$=40mm=front view and parallel to XY.
7. From m'_2 draw a projector. With l as centre and true length =70mm as radius draw an arc to intersect the above projector at m_2.
8. Join lm_2 and measure Ø (Inclination of line with VP).
9. Through m_2 draw the locus of m. With l as centre and 60mm top view as radius draw an arc to cut the locus of m at m. Join lm(top view)

Example: 10

The distance between end projectors of two points A & B is 70mm. Point A is 10mm above HP and 15mm in front of VP. Point B is 40 mm in front of VP and 50 mm above HP. Find the length of the line and inclinations with reference planes. Draw the projections of the line.



Steps:

1. Draw a horizontal line XY as the reference line.
2. Locate a point 10 mm above line XY and mark as a'.
3. Mark point a 15 mm below XY on the vertical projector through a' as plan of point A.
4. Draw a projector 70mm from the projector of point A as the distance between end projectors is 70mm
5. Draw an arbitrary horizontal line parallel to XY, which is 50 mm above which represents locus of b' in VP and it cuts the second projector at b'
6. Draw an arbitrary horizontal line parallel to XY, which is 40 mm below it which represents locus of b in HP and it cuts the second projector at b.
7. With a as centre and ab as radius draw an arc to intersect the horizontal through a at b_2. From b_2 draw a projector to intersect the locus of b' at b_2'. Now a' b_2' is the true length of the line AB.
8. With a' as centre and a'b' as radius draw an arc to intersect the horizontal through a' at b'_1. From b'_1 draw a projector to intersect the locus of b at b_1. Now a b_1 is the true length of the line AB.

PROJECTIONS OF PLANES

Plane figure has only two dimensions; has no thickness. It may be of any shape, such as triangular, square, pentagonal, hexagonal, circular etc.

Types of planes:

1. Perpendicular planes

2. Oblique planes

Perpendicular planes are those planes which are perpendicular to one or both reference planes. These planes can be divided into the following types

a. Perpendicular to one reference plane and parallel to the other.
b. Perpendicular to both reference planes.
c. Perpendicular to one reference plane and inclined to the other.

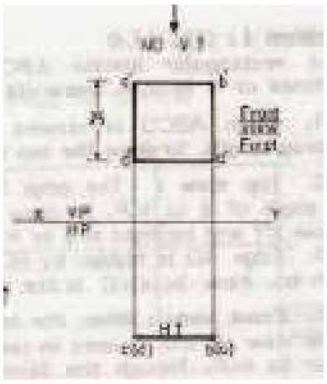
Oblique Planes are those planes which are inclined to both reference planes.

When a plane is parallel to a reference plane the projection of the plane on that reference plane is of true shape of the plane. If the plane is parallel to HP the top view is the true shape of the plane. If the plane is parallel to VP the front view is of true shape of the plane.

When a plane is perpendicular to a reference plane the projection of the plane on that reference plane is a straight line. If the plane is perpendicular to HP the top view is a straight line and if the plane is perpendicular to VP the front view is a straight line.

Example: 1

A square lamina ABCD of side 25mm each is perpendicular to HP and parallel to VP. Draw the projections.

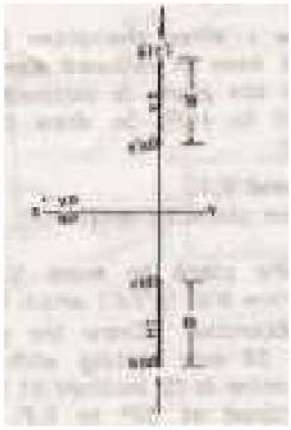


Steps:

1. Since the plane is parallel to VP the front view is the true shape and size i.e. square shape of side 25mm each. The front view a'b'c'd' is drawn above XY as a square of side 25mm.
2. When the plane is viewed from the top, edge BC is seen in true length and parallel to VP. So draw the top view bc (true length) parallel to and below XY. In the top view ad coincides with bc and is marked as (a)(d.)

Example:2

A square lamina ABCD of side 30mm each is perpendicular to both HP and VP. Draw the projections.

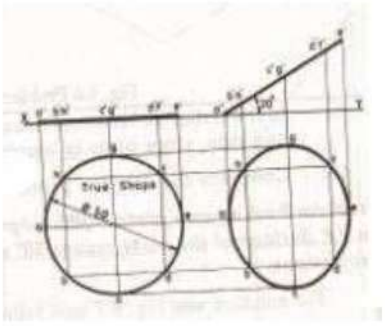


Steps:

1. In front view edge AB is seen in true length. It is above and perpendicular to XY. Draw its front view a'b'.
2. Edge dc is hidden by AB and its front view d'c' coincides with a'b'
3. In top view edge BC is seen in true length. Draw the top view bc below and perpendicular to XY.
4. a and d are hidden. So in top view mark them as (a)(d) coinciding with bc.

Example: 3

A circular plane of 50mm diameter is resting on HP on one of its points of the periphery with surface of the plate perpendicular to VP and inclined to HP by 30^0. Draw the projections.

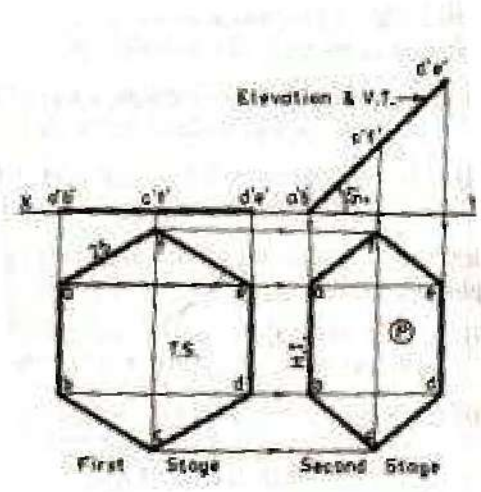


Steps:

1. Draw XY line.
2. Assuming circular plane lying on HP, so the top view will be a circle of same size and its elevation will be a straight line on XY. Draw the top view a circle of diameter 50mm. Project the elevation on XY. Divide the circle into 8 equal parts and mark equal spaced 8 points on the periphery.
3. Rotate the elevation by 30^0 with XY as the plane is inclined to HP.
4. Draw projectors from all marked points of the elevation and horizontal line of the corresponding points from the top view at previous stage to get new top view by intersections. Join them by smooth curves to get the top view.

Example: 4

A regular hexagonal plane, 25mm side each is resting on HP on one of its sides. The surface is perpendicular to VP and inclined to HP by 45^0. Draw its projection.



Steps:

1. Draw the XY line.
2. Since the plane is perpendicular to VP the front view will be a line.
3. Assuming the plane parallel to HP, draw the top view as a regular hexagon abcdef of side 25 mm each with one of its edge ab perpendicular to VP, since the surface is resting on HP on one side.
4. Draw the elevation a'b'c'd'e'f' by projecting this true shaped top view.
5. Rearrange the elevation to new position a'b'c'd'e'f' keeping a'b' on XY line and straight line elevation making 45^0 with XY.
6. Draw projectors from all points on the elevation and draw horizontal lines from corresponding point on the top view in the 1st stage to get the new top view by intersections. Join them to get the top view. abcdef.
7. Elevation and plan in the 2nd stage are the projections of the plane.

PROJECTION OF SOLIDS

Solids

A 3-D object having length, breadth and thickness and bounded by surfaces which may be either plane or curved, or combination of the two.

- Classified under two main headings
  - Polyhedron
  - Solids of revolution

❖ **Polyhedra**

A solid bounded by planes called faces.

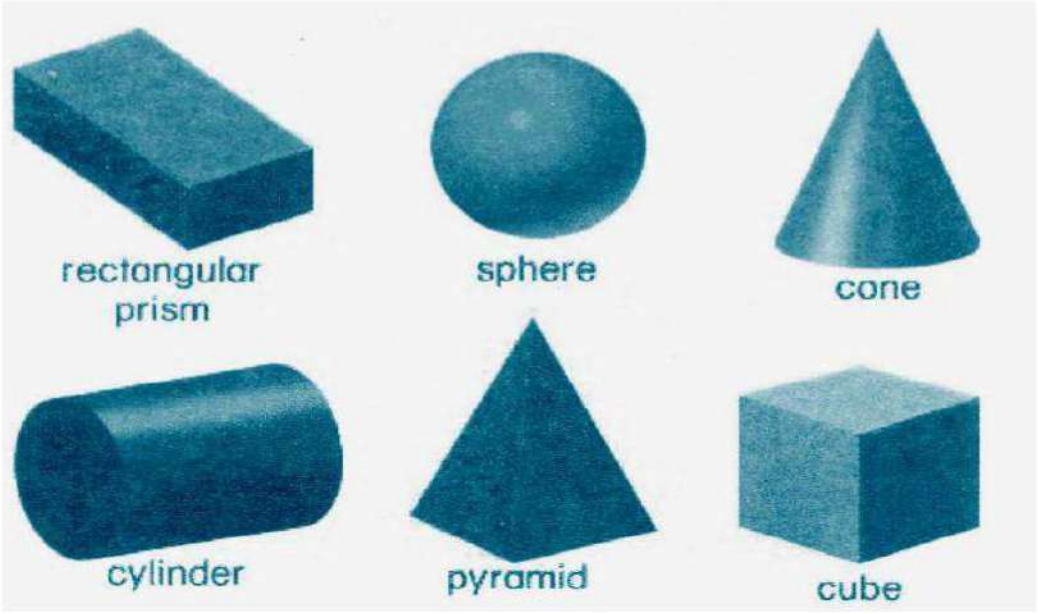
- **Tetrahedron**, **Cube**, **Octahedron**, **Dodecahedron**, **Icosahedrons**
  When all the faces are equal in shape and size, the polyhedral is said to be regular
- **Prism** – A polyhedron having two equal faces called bases or ends, parallel to each other and joined by faces which are parallelograms.
  A right and regular prism has its axis perpendicular to the bases and the bases are regular polygons. All faces are rectangles and are perpendicular to the bases
- **Pyramid** – A polyhedron having a plane as its base and a number of triangular faces meeting at a point called the vertex or apex.
  A right and regular pyramid has its axis perpendicular to the base and the base is is a regular polygon. All faces are isosceles triangles

Prisms and pyramids are named according to the shape of their bases.

❖ **Solids of revolutions**

A solid generated by revolution of a plane about an axis

- **Cylinder** – A right circular cylinder is a solid generated by the revolution of a rectangle about one of its sides
- **Cone** – A right circular cone is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides
- **Sphere** – A solid generated by the revolution of a semi-circle about its diameter
- **Frustum** – A solid obtained by cutting a pyramid or a cone by a plane parallel to its base
- **Truncated** – A solid obtained by cutting a pyramid or a cone by a plane inclined to its base.

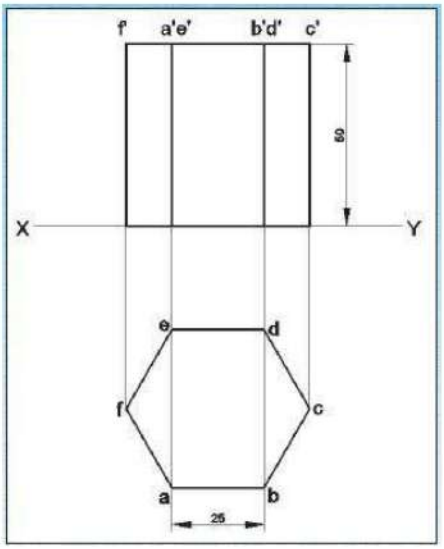


PROJECTION OF SOLIDS WHEN ITS AXIS PERPENDICULAR TO ONE REFERENCE PLANE AND PARALLEL TO THE OTHER

Case (1) Axis perpendicular to the H.P and Parallel to the V.P

EXAMPLE:-1

Project the front view and top view of a hexagonal prism of 25 mm base edges and 50 mm height, having two of its vertical rectangular faces parallel to V.P; and its base resting on H.P.



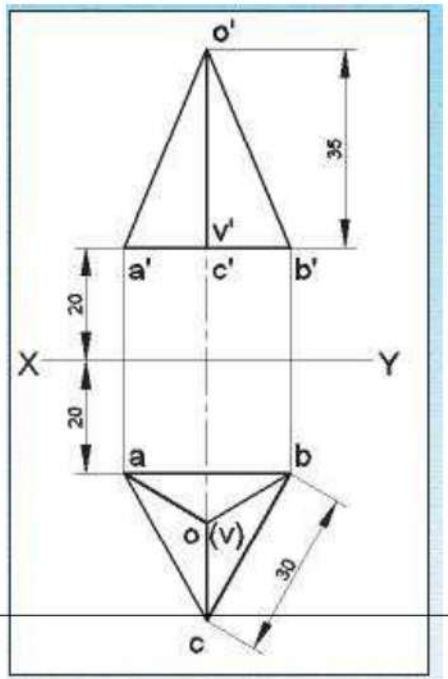
Step:

1. As the axis is perpendicular to HP the top view is hexagonal (i.e. the shape of the base top) and the hexagon abcdef ($a_1b_1c_1d_1e_1f_1$) is to be drawn with two sides parallel to XY representing faces of the prism in the top view.

2. Draw the projectors from the points of the top view and mark $a'_1b'_1c'_1d'_1e'_1f'_1$ on XY. Draw a horizontal line at height 50mm, since the height is 50mm and draw projectors from the $a'_1b'_1c'_1d'_1e'_1f'_1$points till intersects the line. Mark the corresponding points as a'b'c'd'e'f'.

EXAMPLE:-2

A triangular pyramid with 30 mm edges at its base and 35 mm long axis resting on its base with an edge of the base near the V.P, parallel to and 20 mm from the V.P; Draw the projections of the pyramid, if the base is 20 mm above the H.P



Steps:

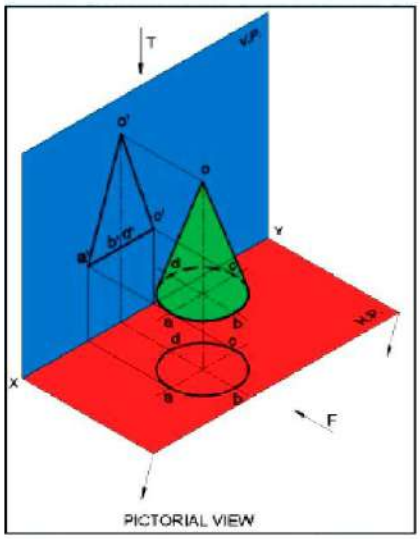
1. Draw the top view of the base abc an equilateral triangle of side 30mm each with ab parallel to XY and 20mm from XY. Find the centre of the triangle as o by drawing perpendicular bisectors of any two sides. Join oa, ob, and oc.
2. Draw the front view of the base as a'c'b' on XY.. Mark o' the front view of o at 35mm above XY on the projectors from o, as the height is 35mm.
3. Join o' with a', b' and c' to get the front view.

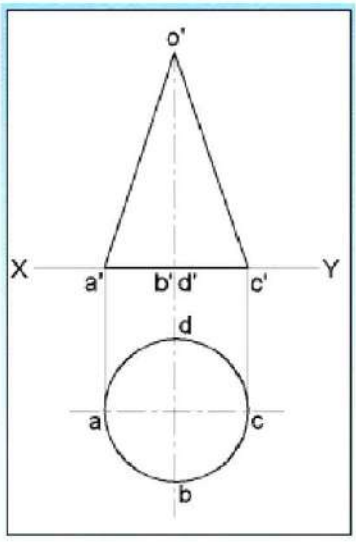
Example: 3

Draw the projection of a right circular cone resting on HP

Steps:

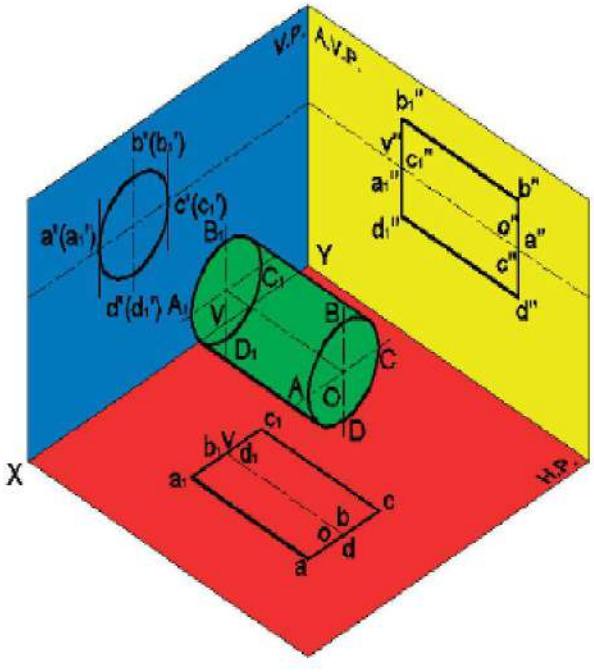
1. Top view gives the true size of the base i.e. a circle. Draw a circle of given diameter. Mark 8 points at equal distance on the periphery. Mark the centre of the circle.
2. Draw projectors from all points on the periphery and mark on XY as front view a'b'c'd'e'f'g'h' . Draw the front view of the centre o' at height above XY. Join o' and other points of the base to get the front view.

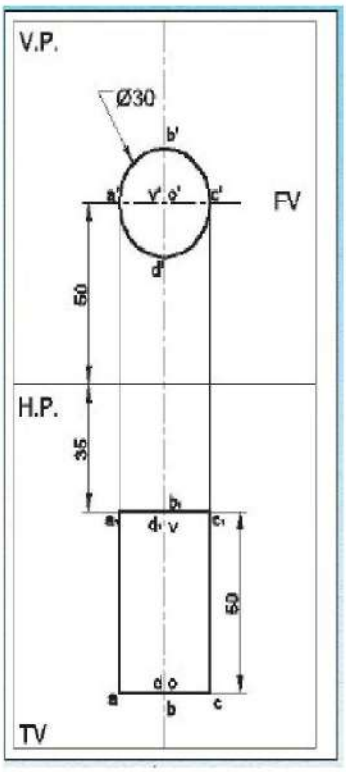




PROJECTION OF SOLIDS WHEN ITS AXIS PERPENDICULAR TO ONE REFERENCE PLANE AND PARALLEL TO THE OTHER

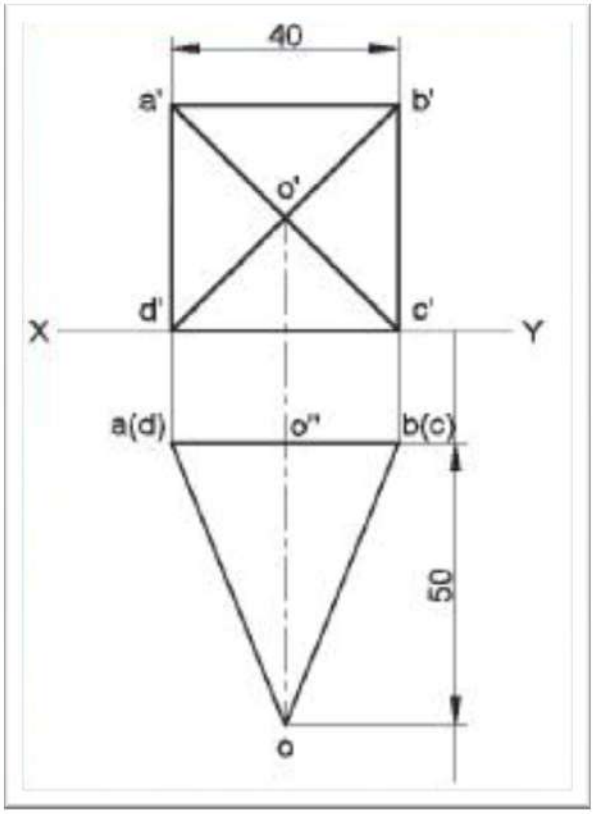
Case (2) Axis perpendicular to the V.P and Parallel to the H.P





EXAMPLE:-4

Draw the front view and top view of a square pyramid of base edge 40 mm and axis 50 mm long resting on HP and axis perpendicular to V.P. The vertex is in front

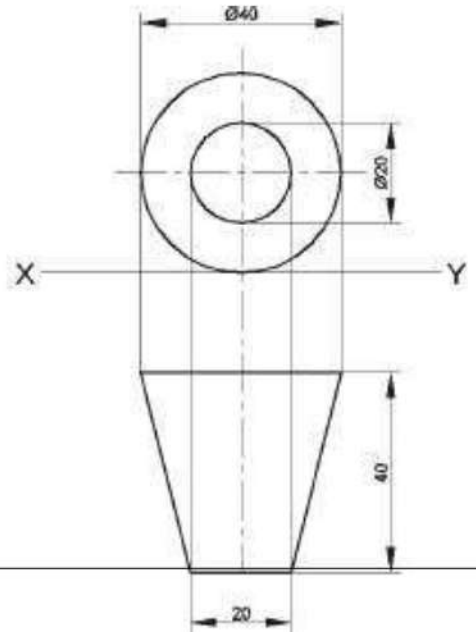


Steps:

1. Since the axis is perpendicular to VP the front view is true size of the base.
2. Draw the top view of the a'b'c'd' with c'd' on XY. Draw the diagonal a'c' and b'd' to get the front view of the apex o'.
3. Draw projector below XY and draw the top view a(d) and b(c) at any distance from XY.
4. Draw the apex o at 50mm below ab on the projector from o'. Join oa and ob to get the top view.

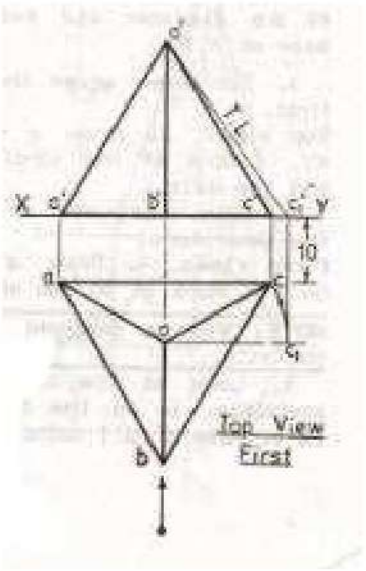
EXAMPLE:-5

The frustum of the cone of 40 mm base diameter and 20 mm cut face diameter, rests on H.P with its 40 mm long axis parallel to H.P and at right angles to V.P, the cut face is in front. Project its **front view** and **top view**



Example: 6

A tetrahedron of side 40mm rests on HP. Draw the projections when one of its edges parallel to and 10mm in front of VP.



Steps;

1. Draw abc equilateral triangle with side 40mm each with ac parallel to and 10 mm below XY as the top view is the true size of base.
2. Draw the centre of the triangle o (by drawing perpendicular bisectors of any two sides and their intersection point). Join oa, ob and oc to get top view.
3. Draw the front view of base a'b'c' on XY.
4. Since none of the slant edge is parallel to VP no slant edge will be true length. Hence to mark o' turn the top view of any one of the slant edge parallel to XY i.e. o as centre and oc as radius draw an arc to cut the line parallel to XY at c_1. Project c_1 and get c'_1 on XY.
5. With c'_1 as centre and 40mm as radius draw an arc to cut the projector drawn from o at o'.
6. Draw a'b'c'.o'a',o'b' and o'c' to get the front view