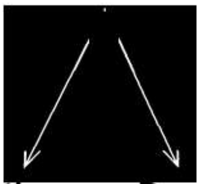
DYNAMICS

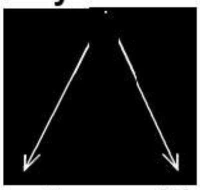
Mechanics



Statics **Dynamics**

In contrast to statics, that deals with rigid bodies at rest, in dynamics we consider the bodies that are in motion. For convenience dynamics is divided into two branches called Kinematics and kinetics.

Dynamics



Kinematics **Kinetics**

Kinematics :
kinematics is concerned with the space time relationship of given motion of a body without reference to the force that cause the motion.

Example : When a wheel rolls along a straight level track with uniform speed, the determination of the shape of the path described by a point on its rim and of the position along with path that the chosen point occupy at any given instant are problems of kinematics.

Kinetics : Kinetics is concerned with the motion of a body or system of bodies under the action of forces causing the motion.

Example : 1) When a constant horizontal force is applied to a body that rests on a smooth horizontal plane, the prediction of the way in which the body moves is a problem of kinetics.

2) Determination of the constant torque that must be applied to the shaft of a given rotor in order to bring it up to a desired speed of rotation in a given interval of time is a problem of kinetics.

Particle :

The whole science of dynamics is based on the natural laws governing motion of a particle or particles. A particle is defined as a material point without dimensions but containing a definite quantity of mater.

But in true sense of term there can be no such thing as a particle, since a definite amount of matter must occupy some space. However, when the size of a body is extremely small compared with its range of motion, it may in certain cases, be considered as a particle.

Example : 1) Star & planets, although may thousands of kilometer in diameter are as small compared with their range of motion that they may be considered as particles in space.

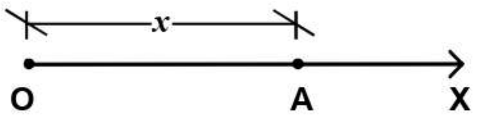
2) The dimensions of a rifle bullet are so small compared with those of its trajectory that ordinarily it may be considered as a particle.

Whenever a particle moves through space, it describes a trace that is called the path. Which may be either a space curve / tortuous path or a plane curve / plane path. In the simplest case when the plane path is a straight line the particle is said to have rectilinear motion or else, it is a curvilinear motion.

Displacement, velocity and acceleration function

These are the three quantities that are necessary to completely describe the motion of a particle in dynamics.

Displacement :

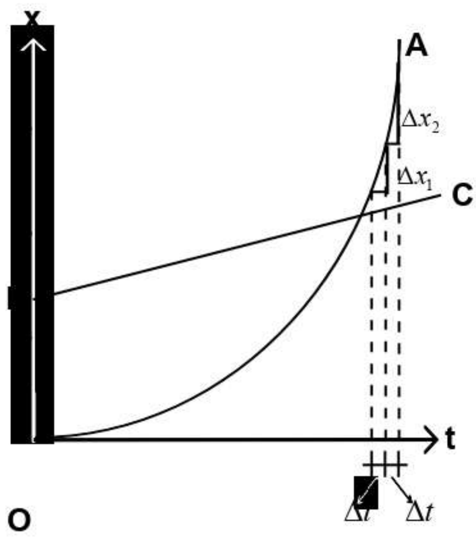


Assuming the motion of the particle along X-Axis, we can define the displacement of a particle by its x-coordinate, measured from the fixed reference point O. This displacement is considered as positive when the particle is to the right of the origin O and as negative when to the left.

As the particle moves, the displacement varies with time, and the motion of the particle is completely defined if we know the displacement x at any instant of time t. Analytically, this relation can be expressed by the general displacement – time equation, x= f(t), where f(t) stands for any function of time. This equation will take different forms depending upon how the particle moves along the x-axis. The displacement is measured by the unit of length.

Velocity :

In rectilinear motion, considering the case of uniform rectilinear motion, as represented graphically by the straight line BC, we see that for equal intervals of time Δt, the particle receives equal increments of displacement Δx. This velocity v of uniform motion is given by the equation v = Δx/ Δt. this velocity is considered as positive if the displacement x is increasing with time and negative if it is decreasing with time.



In the more general case of non-uniform rectilinear motion of a particle as represented graphically by the curve OA, in equal intervals of time Δt, the particle receives unequal increments of displacement. Δx_1 & Δx_2, Thus, if Δx denotes the increment of displacement received during the interval of time Δt, the average velocity during this time is given by the equation v_{av}= Δx / Δt. As the time interval Δt is taken smaller and smaller, the motion during the interval becomes more & more nearly uniform so that this ratio $\Delta x \,/\, \Delta t$ approaches more and more closely to the velocity at any particular instant of time interval. Taking the limit approach we obtain the instantaneous velocity of the particle $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \dot{x}$.

Thus we see that the velocity time relationship of a moving particle can always be obtained by differentiating the displacement time equation. If the derivative is (+)ve i.e. the displacement increases with time, the velocity is positive and has a positive direction of the x-axis other wise it is negative. The velocity is measured by the unit of length divided by the unit of time.

Acceleration :

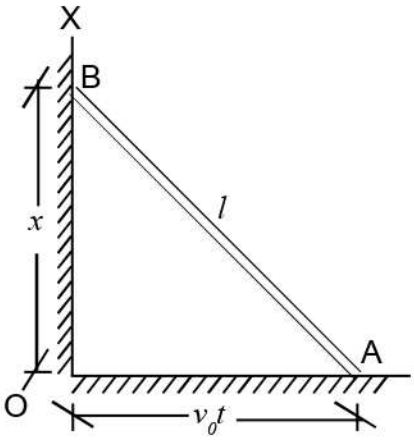
If the rectilinear motion of particle is non uniform its velocity is changing with time and we have acceleration. But in case of uniform motion, the velocity remains constant and there is zero acceleration. When the particle receives equal increments of velocity “v in equal intervals of time Δt, we have motion with constant acceleration as given by the equation $a = \frac{\Delta v}{\Delta t}$. Acceleration is consider positive when the velocity obtains positive increments in successive intervals of time and negative if the velocity is decreasing. However the acceleration of a particle may be positive when its velocity is negative of vice versa.

It the more general case when in equal intervals of time Δt, the increments of velocity Δv_1 and Δv_2 are unequal, we have a motion of the particle with variable acceleration. To obtain the acceleration of the particle at any instant t in such a case, we take the limit approach using the equation

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d^2 x}{dt^2} = \ddot{x} \ .$$

Thus, in any case of rectilinear motion of a particle, the acceleration time relationship can be expressed analytically by taking second derivative with respective time of the displacement time equation. The acceleration is measured by the unit of length divided by square of unit of time.

Problem 1:



A slender bar AB of length l which remains always in the same vertical plane has its ends A & B constrained to remain in contact with a horizontal floor and a vertical wall respectively as shown in the figure. The bar starts from the vertical position and the end A is moving along the floor with constant velocity v_0 So that its displacement OA = v_0t. Write the displacement – time, velocity-time and acceleration-time equations for the vertical motion of the end B of the bar.

Solution :

Let us choose x-axis along the vertical line of motion of end B, i.e. along OB with origin at O.

From the geometrical relationship of the figure, the displacement x of the end b is given by the equation.

$$x = \sqrt{l^2 - (v_0 t)^2} \text{ ------------- (1)}$$

which is the desired displacement time equation.

Differentiating w.r.t. time once, we get

$$\dot{x} = (-)\frac{v_0^2 t}{\sqrt{l^2 - v_0^2 t^2}} \text{ ---------- (2)}$$

the velocity time equation.

Differentiating w.r.t. time twice, we get

$$\ddot{x} = (-)\frac{v_0^2 l^2}{\left(l^2 - v_0^2 t^2\right)^{3/2}} \text{ ---------- (3)}$$

the acceleration time equation.

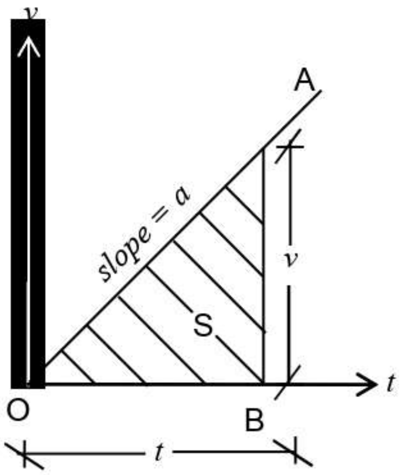
These expressions are valid in the interval $(0 < t < l/v_0)$

Problem 2 :

A particle starting from rest moves rectilinearly with constant acceleration a and acquires a velocity v in time t, traveling a total distance s. Develop formulae showing the relationship that must exist among any three of these quantities.

Solution :

Let us draw the velocity time diagram. Here



i) The acceleration a is represented by the slope of the straight line OA.

ii) The velocity v by the ordinate BA.

iii) The time t by the abscissa OB.

iv) The distance traveled s by the area OAB.

From geometry of the figure we may write $v = at$ ------ (1), $s = 1/2\ vt$ ------- (2)

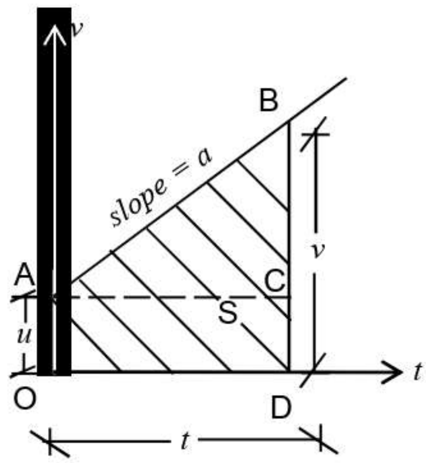
Putting (1) in (2), $s = 1/2\ at^2$ -------- (3)

Eliminating t from equations (1) & (2), $t = \frac{v}{a}$ & $s = \frac{1}{2}v.\frac{v}{a}$ or $v = \sqrt{2as}$ ---------- (4)

But these formulae do not account for any initial displacement or initial velocity.

Problem 3 :

A particle moving with initial velocity u moves with constant acceleration a and acquires a velocity v after time t during which it travels a total distance s. Develop formulae showing relationship between these four quantities. Also give the expression for displacement, if it has an initial displacement s_0 before it starts moving.



Solution :

Acceleration a = slope of AB

Initial velocity u = OA along velocity axis.

Velocity v = ordinate BD

Time t = ordinate OD

Distance s = area of OABCD

From the geometry of the figure, we may write,

v = CD + CB = OA + CB = $u + at$ --------- (1)

s = $\triangle$ ABC + $\square$ OACD = $ut + 1/2\, at^2$ ----------- (2)

Eliminating t, $t = \frac{v-u}{a}$, $s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a \times \left(\frac{v-u}{a}\right)^2 = \frac{uv-u^2}{a} + \frac{(v-u)^2}{2a}$

$$= \frac{2uv - 2u^2 + v^2 + u^2 - 2uv}{2a} = \frac{v^2 - u^2}{2a}$$

or $v^2 - u^2 = 2as$ ---------- (3)

When there is initial displacement, $s = s_0 + ut + 1/2\, at^2$.

Principles of dynamics

There are several axioms called the principle of dynamics which are concerned with the relationship between the kind of motion of a particle and the force producing it. These axioms are in fact, broad generalization of Kepler's observation on the motion of heavenly bodies and of carefully conducted experiment with the motion of earthly bodies. The first reliable experiments were made by Galileo in this connection who discovered the first two laws of motion of a particle. However, the complete set of principles and their final formulation were made by Newton after the name of which they are commonly called the Newton's laws of motion.

Newton's Laws of Motion :

First Law : This law also some times called the law of inertia is stated as "Every body continues in a state of rest or of uniform motion in a straight line unless and until it is acted upon by a force to change that state. For practical problems, we consider the surface of the earth as immovable and refer the motion of the particle with respect to the earth i.e. surface of the earth is taken as inertial frame of reference. But for the motion of heavenly bodies (planets & satellites), a system of coordinates defined by stars is taken as the immovable system of reference and their motion with respective these stars considered. From the first law it follows that any change in the motion of a particle is the effect of a force, that form the concept of force. However, the relation between this change in velocity and the force that produces it given by the second law of dynamics.

Second Law :

The observations made by Galileo on the basis of his experiments on falling bodies and bodies moving along inclined planes, was verified by experiment on pendulums of various materials by Newton and generalized as second law of dynamics which states that the acceleration of a given particle / body is proportional to the force applied to it and acts in the direction of force.

In this law, as formulated above, there is no mention of the motion of the particle before it was acted upon by the force and hence the acceleration of a particle produced by a given force is independent of the motion of the particle. Thus a given force acting on particle produces the same acceleration regardless of whether the particle is at rest or in motion and also regardless of the direction of motion.

Again, there is no reference as to how many forces are acting on the particle. So if a system of concurrent forces is acting on the particle, then each force is expected to produce exactly the same acceleration as it would have if acting alone.

Thus, the resultant acceleration of a particle may be obtained as the vector sum of the accelerations produced separately by each of the forces acting upon it. Moreover, since the acceleration produced by each force is in the direction of the force and proportional to it, the resultant acceleration act in the direction of, and proportional to, the resultant force.

By using Newton's Second law, the general equation of motion of particle can be established as follows :

Let the gravity force i.e. weight of the particle W, acting alone produces an acceleration equal to g.

It instead of weight W, a force F acts on the same particle, then from the 2nd Law it follows that the acceleration a produced by this force will act the direction of the force and will be in the same ratio to the gravitational acceleration g as the force F to gravity force W.

$$\Rightarrow \frac{a}{g} = \frac{F}{W} \text{ or } \frac{W}{g} = \frac{F}{a} \text{ or } F = \frac{W}{g}a$$

This is the general equation of motion of a particle for which its acceleration at any instant can be obtained if the magnitude of force F is known. Also it is evident that for a given magnitude of force f, the acceleration produced is inversely proportional to the factor W/g, which is called mass of the particle denoted by m. This factor measure the degree of sluggishness (inherent resistance against motion) with which the particle yields (responds) to the action of applied force and is a measure of the inertia of the particle. Thus the second law of dynamics forms the basis of concept of mass. Thus using the notation $m = W/g$, the general equation of motion of a particle becomes $F=ma$.

Third Law :

By using the first two laws formulated above the motion of a single particle subjected to the action of given forces can be investigated. However in more complicated cases where it was required to deal with a system of particles or rigid bodies the mutual actions & reactions among them must be taken into account, which is given by Newton's third law. It states that "To every action there is always an equal and opposite reaction or in other words, the mutual action of any two bodies are always equal and oppositely directed".

This implies that if one body presses another, it is in turn is pressed by the other with an equal force in the opposite direction and if one body attracts another from a distance, this other attracts it with an equal and opposite force.

Example :

Sun attracts the earth with a certain force and therefore the earth attracts the sun with exactly the same force.

This holds good not only for the forces of gravitation but also for the other kind of forces such as magnetic or electrical forces. Thus a magnet attracts a piece of iron no more than the iron attracts the magnet.

The equation of motion can be used so solve two kinds of problems.

1) Motion of particle is given, to find the force necessary to produce such a motion.

2) The force is given and it is required to find the motion of the particle.

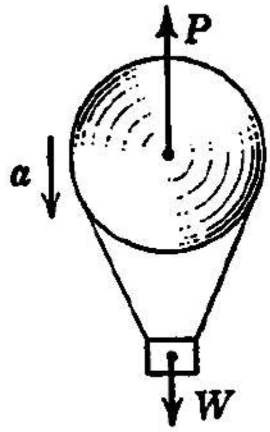
Let us discuss the first kind of problem :

Problem 4 :

A balloon of gross weight W falls vertically downward with constant acceleration a. What amount of ballast Q must be thrown out in order to give the balloon an equal upward acceleration a. Neglect air resistance.

Solution :

Case 1 : When the balloon is falling



The active forces on the balloon are :

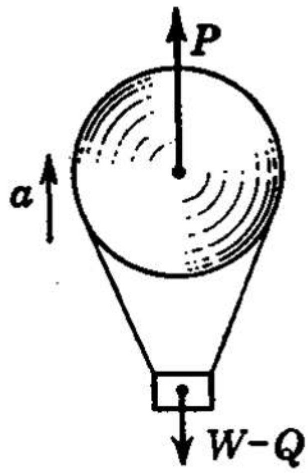
i) The total weight 'W' including the ballast, that acts vertically down ward and.

ii) The buoyant force P, acting upward representing the weight of the volume of air displaced.

The equation of motion in this case may be written as

$$\frac{W}{g}a = W - P \quad \text{----------------- (1)}$$

Case II : When the balloon in rising :



The active forces on the balloon are :

i) The balance weight (W-Q) excluding the ballast acting vertically downward.

ii) The same buoyant force P acting upward as in the previous case.

The equation of motion in this case may similarly be written as :

$$\frac{W}{g}a = P - (W - Q) \quad \text{------------- (2)}$$

Solving equations (1) & (2), we obtain

$$Q = \frac{2Wa}{g+a} \quad \text{------------------------- (3)}$$

Particular cases :

i) When a=0, if the balloon was in equilibrium, Q= 0 & W=P

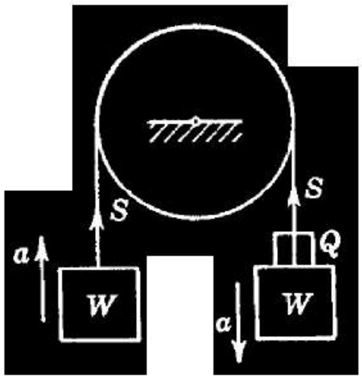
$\Rightarrow$ No ballast would have to be thrown out to rise with the same zero acceleration.

ii) When a=g, if the balloon was falling freely, Q=W & P=0

$\Rightarrow$ It is impossible to throw out sufficient ballast to cause the balloon to rise.

Problem 5 :

Two equal weights W and a single weight Q are attached to the ends of the flexible but in extensible cord over hanging a pulley as shown in the figure. If the system moves with constant acceleration a as indicated by arrows, find the magnitude of the weight Q. Neglect air resistance and the inertia of the pulley.



Solution :

Here we have a system of two particles for which there are two equations of motion.

For the weight on the left :

The active forces are the tension in the cord 'S' acting upward and the weight of the body 'W' down ward.

The equation of the motion in this case may be written as

$$\frac{W}{g}a = S - W \quad \text{------------------ (1)}$$

For the weight on the right :

The active forces are the same tension in the cord 'S' acting upwards but a down ward force of (W+Q).

The equation of motion in this case may similarly be written as :

$$\left(\frac{W+Q}{g}\right)a = (W+Q) - S \quad \text{------------- (2)}$$

Solving the equations (1) & (2), we have

$$Q = \frac{2Wa}{g-a} \quad \text{----------------------- (3)}$$

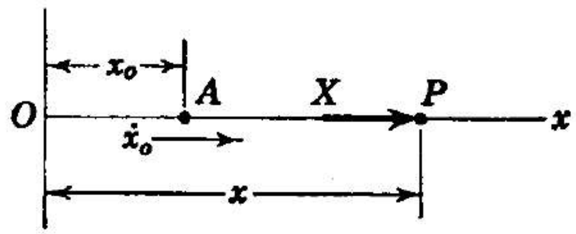
Particular Case : (Interpretation of the solution of the problem)

To produce an acceleration a=g of the system would require a weight Q = ∞, which implies that it is impossible for the weight on the RHS to attain practically a free fall condition.

Motion of a particle acted upon by a constant force :

Now, let us discuss the second kind of dynamics problem, in which the acting force is given and the resulting motion of the particle is required. To start with, let us consider the simplest case of a particle acted upon by a constant force, the direction of which remains unchanged. In this case, the particle moves rectilinearly in the direction of force with a constant acceleration.

Let the line of motion be along x-axis and the magnitude of force be denoted by X. Also the particle starts from A with initial displacement x_0, when t = 0.



The equation of motion can be written as :

$$X = \frac{W}{g}a = \frac{W}{g}\ddot{x} \text{ or } \ddot{x} = \frac{X}{W/g} = a = \frac{d^2x}{dt^2} \text{ ----------------(1)}$$

To find the velocity $\dot{x}$ and displacement x as function of time, integrating this differential equation,

we get, $d\left(\frac{dx}{dt}\right) = d(\dot{x}) = a dt$ or $\frac{dx}{dt} = \dot{x} = at + C_1$, where C_1 is the constant of integration.

Boundary condition :

At t = 0 , $\dot{x} = \dot{x}_0 \Rightarrow C_1 = \dot{x}_0$

$\therefore\ \dot{x} = \dot{x}_0 + at$ -----------(2)

This is the general velocity time equation for the rectilinear motion of a particle under the action of a constant force X.

Writing equation (2) in the form - $\frac{dx}{dt} = \dot{x} + at$ or $dx = \dot{x}_0 dt + atdt$ and integrating again, we obtain, $x = \dot{x}_0 t + \frac{1}{2}at^2 + C_2$ ------------------(3)

Where C_2 again a constant of integration.

Boundary Condition :

At t = 0, $x = x_0 \Rightarrow C_2 = x_0$

$\therefore \quad x = x_0 + \dot{x}_0 t + \frac{1}{2}at^2$

This is the general displacement time equation for the rectilinear motion of a particle under the action of a constant force X.

Thus the initial conditions influence the motion of a particle quite as much as does the acting force. Infect the initial condition represents the heredity of motion, while the acting force represents its environment.

Particular case :

Case – I – Freely falling body :

Acting force X= W and a = g

Then from equations (2) & (3), $\dot{x} = \dot{x}_0 + gt$ ------------------------ (2)'

$x = x_0 + \dot{x}_0 t + \frac{1}{2}gt^2$ ----------------(3)'

Case II – Body starts to fall from rest and from the origin :

In this case $x_0 = 0$ and $\dot{x}_0 = 0$ and the equations (2) & (3) reduces to

$\dot{x} = gt$ -------------------- (2)"

$x = \frac{1}{2}gt^2$ ---------------------(3)"

Problem 6 :

A particle projected vertically upward is at a height h after t_1 seconds and again after t_2 seconds . Find the sight h and also the initial velocity v_0 with which the particle was projected.

Solution :

Neglecting air resistance, the active force in the particle is its own gravity force W which is always directed vertically downward.

Taking x-axis along the vertical line of motion, the origin at the starting point and considering the upward displacement as positive, from displacement time equation we have :

When $t = t_1$, $h = v_0 t_1 - \frac{1}{2} g t_1^2$ ------------------------(1)

& When $t = t_2$, $h = v_0 t_2 - \frac{1}{2} g t_2^2$ ------------------- (2)

Multiply equation (1) with t_2 & equation (2) with t_1 & subtracting

$h(t_2 - t_1) = \frac{1}{2} g t_1 t_2 (t_2 - t_1)$ or $h = \frac{1}{2} g t_1 t_2$ --------------- (3)

Equating equations (1) & (2), $v_0 t_1 - \frac{1}{2} g t_1^2 = v_0 t_2 - \frac{1}{2} g t_2^2$

or $v_0 (t_1 - t_2) = \frac{1}{2} g (t_1 - t_2)(t_1 + t_2)$ or $v_0 = \frac{1}{2} g (t_1 + t_2)$ ----------- (4)

Problem 7 :

From the top of a tower of height h = 40m a ball is dropped at the same instant that another is projected vertically upward from the ground with initial velocity v_0 = 20 m/s. However from the top do they pass and with what relative velocity ?

Solution :

Case 1 : For the ball dropped from the top :

The top of the tower is taken as the origin and downward displacement is considered as positive. The initial displacement and initial velocity are zero. Acceleration due to its own gravitational force is downward. Thus the displacement time equation will be

$$x_1 = \frac{1}{2}gt^2 \text{ ------------------------- (1)}$$

Case 2 : For the ball projected from the ground :

The ground is taken as the origin and the upward displacement is considered as positive. Here initial displacement = 0 and initial velocity = v_0 upward while the acceleration due to gravity is down ward. Thus the displacement time equation will be

$$x_2 = v_0 t - \frac{1}{2}gt^2 \text{ ------------------------- (2)}$$

When the balls pass each other, we must have

$$x_1 + x_2 = h \text{ ---------------------------- (3)}$$

Substituting the values of x_1 and x_2 from equation (1) & (2) in (3), we obtain,

$$\frac{1}{2}gt^2 + v_0 t - \frac{1}{2}gt^2 = h \text{ or } h = v_0 t \text{ ----------------------(4)}$$

Putting numerical values, we have $t = \frac{v_0}{h} = \frac{40}{20} = 2\sec$

Putting this value of t in equation (1) we find

$$(x_1)_{t=2} = \frac{1}{2} \times 9.81 \times 4 = 19.62m.$$

Relative Velocity :

The velocity time equation for case - I,

$$\dot{x}_1 = \frac{dx_1}{dt} = \frac{1}{2}g.2t = gt \downarrow$$

At time t = 2 Sec, Velocity $\dot{x}_1 = 9.81 \times 2 = 19.62m/s \downarrow$

The velocity time equation for case - II,

$$\dot{x}_2 = \frac{dx_2}{dt} = v_0 - gt \uparrow$$

At time t = 2 Sec, Velocity $\dot{x}_2 = 20 - 19.62 = 0.38m/s. \uparrow$

$\therefore$ Their relative velocity $v_R = \dot{x}_1 - \dot{x}_2 = 19.62 - (-0.38) = 20m/s$

Hence the balls pass 19.62 m. below the top of the tower with a relative velocity of 20 m/s. i.e. same as the initial velocity.