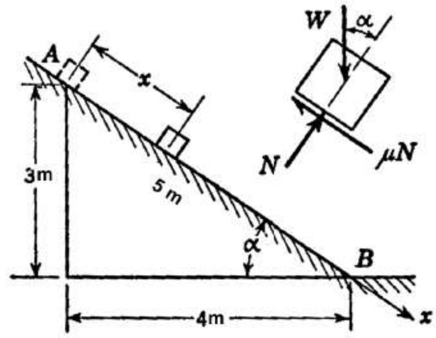
Problem 8 :



A small block of weight W is placed on an inclined plane as shown in the figure. What time interval t will be required for the block to traverse the distance AB, if it is released from rest at A and the coefficient of kinetic friction on the plane is μ=0.3. What is the velocity at B.

Solution :

From the free body diagram of the block, the resultant force along the plane $X = W(\sin\alpha - \mu\cos\alpha)$ acting down ward.

Resulting acceleration $a = \dfrac{X}{W/g} = g(\sin\alpha - \mu\cos\alpha)$

For sliding of the block, a must be positive $\Rightarrow (\sin\alpha - \mu\cos\alpha) > 0$ or $\tan\alpha \geq \mu$

Here $\sin\alpha = 3/5$ and $\cos\alpha = 4/5$ & putting other numerical values

$$a = 9.81\left(\frac{3}{5} - 0.3\times\frac{4}{5}\right) = 3.53 m/s^2$$

Again the initial displacement = 0 and initial velocity =0

The displacement time equation becomes $x = 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2$

Putting numerical valuces $50 = \frac{1}{2}\times 3.53\times t^2$ or $t^2 = 28.32$ or t = 5.32 s

Substituting the value of t in the velocity time equation,

Velocity at B, $\dot{x}_b = 0 + at = at = 3.53\times 5.32 = 18.78 m/s$

D' Alembert's Principle :

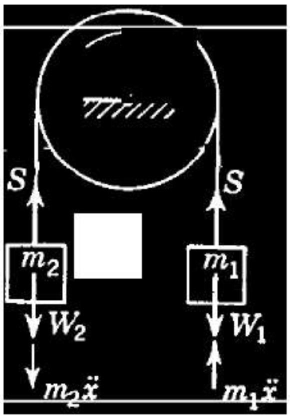
The differential equation of rectilinear motion of a particle $X = m\ddot{x}$ can be written in the form $X - m\ddot{x} = 0$, where x denotes the resultant in the direction of X-axis of all applied forces and m, the mass of the particle.

This equation of motion of particle is of the same form as an equation of static equilibrium and may be considered as an equation of dynamic equilibrium. For writing this equation, we need only to consider in addition to the real forces acting on the particle, an imaginary force $-m\ddot{x}$. This force equal to the product of the mass of the particle and its acceleration and directed oppositely to the acceleration, is called inertia force of the particle.

D' Alembert was first to point out this fact that equation of motion could be written as equilibrium equations simply by introducing inertia forces in addition to the real forces acting on a system. This idea is known as the D' Alembert's principles and has definite advantage in the solution of engineering problems of dynamics.

Due to application of this principle, when dealing with a system having one degree of freedom, we need to write only one equation of dynamic equilibrium in stead of writing as many equation of motion as there are particles. It particularly becomes very useful when used in conjunction with the method of virtual work.

Problem 9 : Two unequal weights W_1 and W_2 are attached to the ends of a flexible but inextensible cord overhanging a pulley as shown in the figure ($W_1 > W_2$). Neglecting air resistance and inertia of the pulley, find the magnitude of acceleration of the weights.



Solution :

Here we have a system of particle connected between themselves and so constrained that each particle can have only a rectilinear motion.

Assuming motion of the system in the direction shown by the arrow on the pulley, there will be upward acceleration $\ddot{x}$ of the weight W_2 and downward acceleration $\ddot{x}$ of the weight W_1. Denoting the masses by m_1 and m_2 respectively, the corresponding inertia forces act as shown in the figure. Assuming the tension in the string (String Reaction) to be S on both sides of the pulley, neglecting friction on the pulley, we can have a system of forces in equilibrium for each particle by adding the inertia forces to the real forces.

$S - W_2 - m_2\ddot{x} = 0$ ----------------- (1)

$W_1 - S - m_1\ddot{x} = 0$ ----------------- (2)

Eliminating S from these equations, $W_2 + m_2\ddot{x} = W_1 + m_1\ddot{x}$ ------------- (3)

Again this problem can be conceived in another way i.e. since each particle is in equilibrium we can say that the entire system of forces is in equilibrium. So instead of writing separate equations of equilibrium for each particle, we can write one equation equilibrium for the entire system by equating to zero the algebraic sum of moment of all the forces (including the inertia forces) with respect of the axis of the pulley.

In this case, we need not consider the internal forces or reaction S of the system and can directly write.

$$(W_2 + m_2\ddot{x})r = (W_1 - m_1\ddot{x})r \text{ or } W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

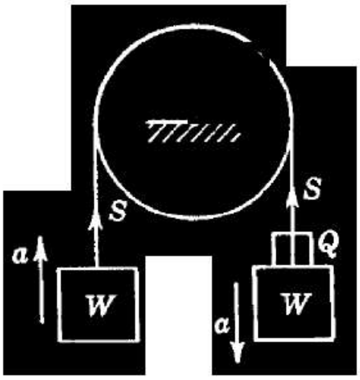
from which $\ddot{x} = \left(\dfrac{W_1 - W_2}{W_1 + W_2}\right)g = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)g$

Problem 10 :

Let us solve the previous problem No. 5 by D' Alembert's Principle.

Solution :

Here also we have a system of two particles so connected that it has single degree of freedom.



Taking moment about the centre of the pulley O, we have $\left(W + \dfrac{W}{g}a\right)r = \left\{(W+Q) - \dfrac{(W+Q)}{g}a\right\}r$

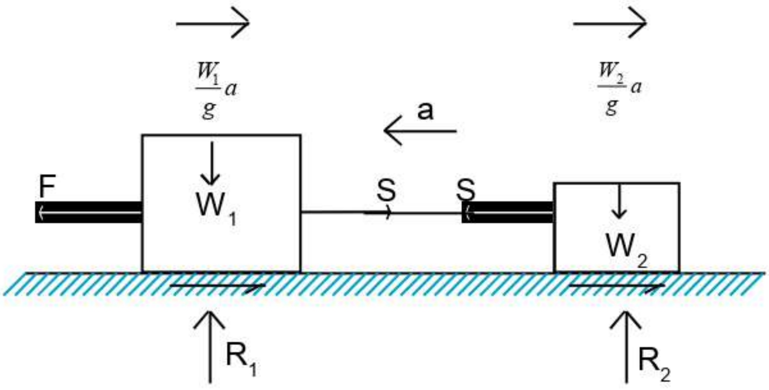
or $W + \dfrac{W}{g}a = (W+Q) - \dfrac{(W+Q)}{g}a$

or $\dfrac{W}{g}a = Q - \dfrac{Wa}{g} - \dfrac{Qa}{g}$ or $\dfrac{2Wa}{g} = Q\left(1 - \dfrac{a}{g}\right)$

or $\dfrac{2Wa}{g} = Q\left(\dfrac{g-a}{g}\right)$ or $Q = 2\dfrac{Wa}{(g-a)}$

Problem 11 :

Two bodies of weight W_1 & W_2 are connected by a thread and move along a rough horizontal plane under the action of a force F applied to the first body. If the coefficient of friction between the sliding surface of bodies and the plane is 0.3, determine the acceleration of the bodies and tension in the thread.



Given m_1 = 80 Kg., m_2 = 20 Kg., μ = 0.3, F= 400 N.

Solution :

Here also we have a system of two particles so connected that, it has a single degree of freedom. So we can apply D' Alembert's principle to the system and write only one equation of equilibrium for the entire system taking only the active forces but without considering the internal forces (reactions).

Along horaizontal direction, $F = \frac{W_1}{g}a + \frac{W_2}{g}a + \mu W_1 + +\mu W_2$

$$= \left(\frac{W_1 + W_2}{g}\right)a + \mu(W_1 + W_2)$$

or, $\left(\frac{W_1 + W_2}{g}\right)a = F - \mu(W_1 + W_2)$

$$or, \quad a = \frac{F - \mu(W_1 + W_2)}{\left(\frac{W_1 + W_2}{g}\right)} \qquad or, \qquad a = \frac{F - \mu g(m_1 + m_2)}{(m_1 + m_2)} = \frac{400 - 0.3 \times 9.81(80 + 20)}{(80 + 20)} = 1.06 m/s^2$$

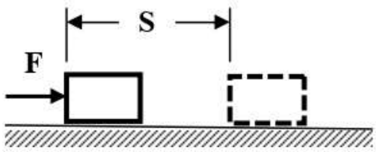
Considering the equilibrium of second body

$$S = \frac{W_2}{g}a + \mu W_2 = W_2\left(\frac{a}{g} + \mu\right) = m_2(a + \mu g) = 20(1.06 + 0.3 \times 9.81) = 80\, N$$

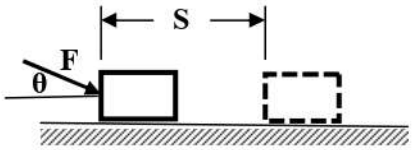
** ** **

6.2 Work, Energy and Power

Work: When a forceis acting on a body and the body undergoes a displacement then some wok is said to be done. Thus the work done by a force on a moving body is defined as *the product of the force and the distance moved in the direction of the force*.



Body moving with direction of force



Body not moving with direction of force

When force F acts on the body and body displaces in the direction of force as shown in fig. The work done by the force F = Force x Distance = F x S

When the force acts on the body at an angle θ to the horizontal and the body moves in horizontal direction by distance s, then work done by the force F = Component of the force in the direction of displacement x distance= Fcosθ x S.

From definition of work it is obvious that unit of work is obtained by multiplying unit of force by unit of length. Hence, if unit of force is newton and unit of distance is meter then unit of work is N.m. So one N-m of work is denoted by one Joule(J). Hence one joule may be defined as *the amount of work done by one newton force when the particle moves one meter in the direction of the force.*

Energy: *Energy is defined as the capacity to do work*. There are many forms of energy like heat energy, mechanical energy, electrical energy, and chemical energy. In mechanics mostly discussed about mechanical energy. The mechanical energy may be classified in to two forms i.e Potential energy and Kinetic energy.

*Potential Energy*is the capacity to do work due to position of the body. A body of weight 'W' held at a height 'h' possesses an energy Wh.

Kinetic Energy is the capacity to do work due to motion of the body. Consider a car moving with a velocity v m/s. If the engine is stopped, it still moves forward doing work against frictional resistance and stops at a certain distance s.

From the kinematic of the motion, $0 - u^2 = 2as$

$$a = -\frac{u^2}{2s}$$

From D'Alembert's principle, $F + \frac{W}{g}a = 0$

$$F - \frac{W}{g} \times \frac{u^2}{2s} = 0$$

$$F = \frac{Wu^2}{2gs}$$

Then, Work done = $F \times s = \frac{Wu^2}{2g}$

This work is done by the energy stored initially in the body.

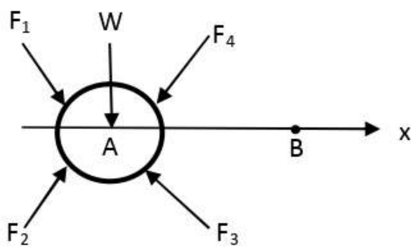
Kinetic Energy= $\frac{1}{2} \times \frac{W}{g} v^2 = \frac{1}{2} m v^2$

Where, m is the mass of the body and v is the velocity of the body.

Unit of energy is same as that of work, since it is nothing but capacity to do work. It is measured in joule (N-m) or kilo Joule (kN-m).

Power: It is defined as the time rate of doing work. Unit of power is watt (w) and is defined as one joule of work done in one second. In practice kilowatt is the commonly used unit which is equal to 1000 watts.

Work Energy equation: Consider the body shown in figure below subjected to a system of forces $F_1, F_2 \ldots \ldots$ and moving with acceleration a in x-direction. Let its initial velocity at A be u and final velocity when it moves distance AB=s be v. Then the resultant of system of the forces must be in x- direction. Let $F = \sum X$



From Newton's second law, $F = \frac{W}{g} a$

Multiplying both side of the equation by elementary distance ds, then

$$Fds = \frac{W}{g}ads = \frac{W}{g}\frac{dv}{dt}ds = \frac{W}{g}dv\frac{ds}{dt}$$

$$Fds = \frac{W}{g}vdv$$

Integrating both sides for the motion from A to B,

$$\int_0^s Fds = \int_u^v \frac{W}{g}vdv$$

$$Fs = \frac{W}{g}\left[\frac{v^2}{2}\right]_u^v = \frac{W}{2g}(v^2 - u^2)$$

Work done = Final kinetic energy- Initial kinetic energy (It is called work energy equation)

This work energy principle may be stated as *the work done by system of forces acting on a body during a displacement is equal to the change in kinetic energy of the body during the same displacement.*

Conservation of Energy:

Conservative force: A force is said to be conservative if the work done by the force on a system that moves between two configurations is independent of the path of the system takes.

*Non-Conservative forces:*A force is said to be non-conservative if the work done by a force on a system depends on the path of the system follows.

Principle of conservation of energy, " when a rigid body or a system of rigid bodies, moves under the action of the conservative forces, the sum of the kinetic energy (T) and the potential energy (V) of the system remains constant.

$T + V = constant$ or $\Delta T + \Delta V = 0$ or $T_1 + V_1 = T_2 + V_2$

Consider a conservative system; say a particle moving from position 1 to position 2.

As the particle moves from position 1 to position 2 it undergoes a change in potential energy,

$\Delta V = V_2 - V_1$

Where: V_1 *and* V_2 are the potential energies in positions 1 and 2 respectively.

As the particle displace from position 1 to 2, let U_{1-2} be the work of the forces that act on the particle. The work U_{1-2} does not depend on the path or the speed with which the particle moves on the path.

Consequently, $U_{1-2} = V_1 - V_2 = -\Delta V$

Then, the principle of work energy $(U_{1-2} = \Delta T)$, yields $-\Delta V = \Delta T$

Where; $\Delta T = T_2 - T_1$ is change in kinetic energy from position 1 to position 2.

Thus relationship may be written as $\Delta T + \Delta V = 0$

$T + V = constant$

Where $T + V$ is called the mechanical energy of the particle.

6.3 Define Momentum & Impulse, Explain Conservation of energy and Linear momentum, Explain collision of elastic bodies and different co-efficient of restitution.

Solved Examples:

Example-1: A pump lifts $40m^3$ of water to a height of 50m and delivers and delivers it with a velocity of 5m/sec. What is the amount of energy spent during the process? If the job is done in half an hour , what is the input power of the pump which has an overall efficiency of 70%?

Solution: Output energy of the pump is spent in lifting $40m^3$ of water to a height of 50m and deliver it with the given kinetic energy of delivery.

Work done in lifting $40m^3$ water to a height of 50m = Wh

Where W=weight of $40m^3$ of water=40 x 9810 N

(Note: $1m^3$ of water weighs 9810 Newton)

Work done= Wh = 40x9810x50 =19620000Nm

Kinetic energy at delivery = $\frac{1}{2} \times \frac{W}{g} \times v^2 = \frac{1}{2} \times \frac{40 \times 9810}{9.81} \times 5^2$ =500000Nm

Total Energy spent= 19620000+ 500000= 20120000Nm= 20120kNm= 20.12 $\times 10^6$ Nm.

This energy is spent by the pump in half an hour i.e. in 30x60=1800sec.

Output power of pump = Output energy spent per second = $\frac{20120000}{1800}$ =11177.8watts=11.1778kW.

Input power= $\frac{Outputpower}{efficiency} = \frac{11.1778}{0.7} = 15.9583kW.$

Example 2: For the same kinetic energy of a body, what should be the change in its velocity if its mass is increased four times?

Answer: Let 'm_1' be the mass of a body moving with a velocity 'v_1'.

Kinetic energy $T_1 = \frac{1}{2} m_1 v_1^2$

When mass is increased four times let the velocity be v_2.

$m_2 = 4 m_1$

$v_2 = ?$

$\therefore T_2 = \frac{1}{2} m_2 v_2^2$

Substitute the value of m_2 in the above equation

$T_2 = \frac{1}{2} 4 m_1 v_2^2$

Given

$T_1 = T_2$

$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} 4 m_1 v_2^2$

$v_1^2 = 4 v_2^2$

$v_2^2 = \frac{v_1^2}{4}$

$v_2 = \sqrt{\frac{v_1^2}{4}}$

$v_2 = \frac{v_1}{2}$

The velocity of the body is halved when its mass is increased four times.

Example 3: Calculate the time taken by a water pump of power 500 W to lift 2000 kg of water to a tank, which is at a height of 15 m from the ground?

Solution: Given $g = 10$ m/s^2

Power of the water pump = 500 W

$P = \frac{W}{t}$

Since the water is lifted through a height of 15 m, work done is equal to the potential energy.

$\therefore \quad P = \frac{mgh}{t}$

Mass of water (m) = 2000 kg

$g = 10 m/s^2$

Height (h) = 15 m

t = ?

$$t = \frac{mgh}{P}$$

$$t = \frac{2000 \times 10 \times 15}{500}$$

$$t = \frac{20 \times 10 \times 15}{5}$$

t = 40 x 15 = 600 s

Time required to lift water = 600 s.

Example 4: A car weighing 1000 kg and travelling at 30 m/s stops at a distance of 50 m decelerating uniformly. What is the force exerted on it by the brakes? What is the work done by the brakes?

solution: Mass of the car (m) = 1000 kg

Initial velocity (u) = 30 m/s

Distance traveled (S)= 50 m

Since the car stops, final velocity (v) = 0

Work done by the brakes = kinetic energy of the car

$$W = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 1000 \times (30)^2$$

$$= 500 \times 900$$

$$= 450000 \text{ J}$$

W = F x S

$$F = \frac{W}{S}$$

$$= \frac{450000}{50}$$

$$= 9000 \text{ N}$$

The force exerted by the brakes is 9000 N.

6.3 Momentum and Impulse

It is clear from the discussion of the previous chapters that for solving kinetic problems, involving force and acceleration, D'Alembert's principle is useful and that for problems involving force, velocity and displacement the work energy method is useful. The Impulse-Momentum method is useful for solving the problems involving force, time and velocity.

Momentum: Momentum is the motion contained in a moving body. It is also defined as the product of an object's mass and velocity. It is a vector.

The SI units of momentum are kg * m/s.

Impulse: Impulse is defined as a force multiplied by the amount of time it acts over. In calculus terms, the impulse can be calculated as the integral of force with respect to time. Alternately, impulse can be calculated as the difference in momentum between two given instances.

The SI units of impulse are N*s or kg*m/s.

Impulse-Momentum Equation: As stated earlier impulse can be calculated as the difference in momentum between two given instances.

If F is the resultant force acting on a body of mass m in the direction of motion of the body, then according to Newton's 2nd law of motion,

$$F = ma$$

But, acceleration, $a = \frac{dv}{dt}$, v is velocity of body.

Then, $F = m\frac{dv}{dt}$

i.e. $Fdt = mdv$

$$\int Fdt = \int mdv$$

If initial velocity is u and after time interval t the velocity becomes v, then

$$\int_0^t Fdt = \int_u^v mdv$$

$$\int_0^t F dt = m[v]_u^v = mv - mu = \textbf{\textit{Impulse Momentum Equation}}$$

The term $\int_0^t F dt$ is called the ***impulse***. If the resultant force is in newton and time in second, the unit of impulse is N*sec.

If the resultant force F is constant during time interval t then, impulse is equal to $F \times t$.

The term $[mv - mu]$ is called change in momentum.

So, Impulse= Change in momentum= Final momentum- Initial momentum.

Impulse momentum equation can be stated as: *The component of the impulse along any direction is equal to change in the component of momentum in that direction*.

Note: Since the velocity is a vector, impulse is also a vector. The impulse momentum equation holds good when the direction of F, u and v are the same.

The impulse momentum equation can be applied in any convenient direction and the kinetic problem involving force, velocity and time can be solved.

Conservation of Linear Momentum:

The impulse momentum equation is: $\int_0^t F dt = mv - mu$

If the resultant force F is zero the equation reduces to $mv = mu$ i.e. Final Momentum = Initial Momentum, Such situation arises in many cases because the force system consists of only action and reaction on the elements of the system. The resultant force is zero, only when entire system is considered.

The principle of conservation of momentum may be defined as: *the momentum is conserved in a system in which resultant force is zero*. In other words, *in a system if resultant force is equal to zero, the initial momentum is equal to final momentum i.e. momentum is conserved.*

Example: When a person jumps off a boat, the action of the person is equal and opposite to reaction of boat. Hence the resultant force of system is zero. If w_1 is weight of the person and w_2 is weight of the boat, v is velocity of the person and the boat before the person jumps out of the boat and v_1, v_2 are the velocities of the person and the boat after jumping. According to conservation of momentum:

$$\frac{w_1 + w_2}{g} v = \frac{w_1}{g} v_1 + \frac{w_2}{g} v_2$$

Similar, equation also holds good when the system of a shell and gun is considered.

Note: It must be noted that conservation of momentum applies to entire system and not to individual elements of the system.

Collision: Collision is a dynamic event consisting of the close approach of two or more bodies resulting in an abrupt change of momentum and exchange of energy.

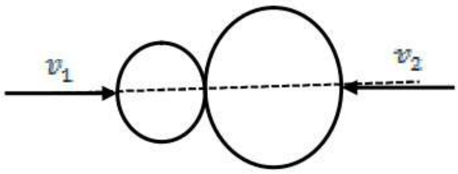
- A collision between two bodies is said to be impact, if the bodies are in contact for a short interval of time and exerts very large force on each other during this short period.
- On impact, the bodies deform first and then recover due to elastic properties and start moving with different velocities.
- The velocities with which they separate depend not only on their velocities of approach but also on the shape, size, elastic properties and line of impact.

Types of Impact:

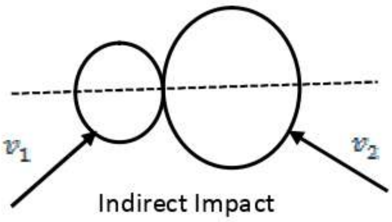
According to Motion of Bodies:

(a) **Direct impact:**If the motion of two colliding bodies is directed along the line of impact (common normal to the colliding surfaces), the impact is said to be direct impact.

(b) **Indirect or Oblique impact:** If the motion of the one or both of the colliding bodies is not directed along the line of impact, the impact is known as oblique impact.



Direct Impact



Indirect Impact

According to properties of bodies:

(a) **Plastic impact:** The material properties of the colliding bodies are plastic or inelastic like putty, so the plastic deformation takes place at point of contact and both bodies move as a single body after impact. In this case only the principle of conservation of momentum hold good.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

(b) **Elastic collision:** The material properties of the colliding bodies are perfectly elastic, so the bodies regain its original shape after impact at the point of contact. In this case both the principle of conservation of momentum and energy hold good.

For conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

For conservation of energy:

$$m_1\frac{u_1^2}{2} + m_2\frac{u_2^2}{2} = m_1\frac{v_1^2}{2} + m_2\frac{v_2^2}{2}$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

From above two equation, we get $u_1 + v_1 = u_2 + v_2$

$$u_1 - u_2 = v_2 - v_1$$

$$Velocity\ of\ aqpproach = Velocity\ of\ separation$$

(c) **Semi elastic collision**: The material properties of the colliding bodies are not perfectly elastic but partly elastic, so the bodies regain part of its original shape after impact at the point of contact. In this case the momentum is conserved but energy partly conserved.

$$v_2 - v_1 = e(u_1 - u_2)$$

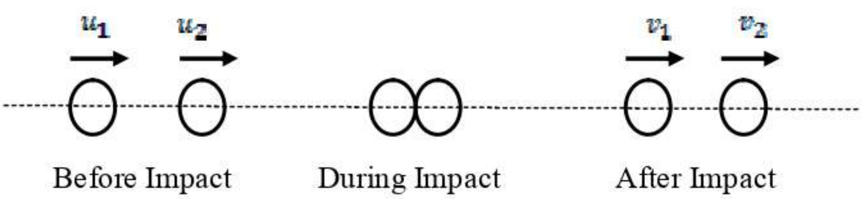
Where: e- coefficient of restitution which value is in between 0 to 1.

Coefficient of Restitution: During the collision the colliding bodies initially under goes a deformation for a small time interval and then recover the deformation in a further small time interval due to elastic property of the body. So the period of collision or time of impact consists of two time intervals i.e. *Period of Deformation* and *Period of Restitution*.

Period of Deformation: It is the time elapse between the instant of initial contact and the instant of maximum deformation of the bodies.

Period of Restitution: It is the time elapse between the instant of maximum deformation condition and the instant of separation of the bodies

Consider two bodies collide and move after collision as shown in Figure below.



Let m_1 –mass of the first body

m_2 –mass of the second body

u_1 –velocity of the first body before impact

u_2 –velocity of the second body before impact

v_1 –velocity of the first body after impact

v_2 –velocity of the second body after impact

Therefore, Impulse during deformation = $F_D dt$

Where, F_D refers to the force that acts during the period of deformation

The magnitude of F_D is varies from zero at the instant of initial contact to the maximum value at the instant of maximum deformation.

Similarly, Impulse during restitution=$F_R dt$

Where, F_R refer to the force that acts during period of restitution.

The magnitude of F_R is varies from maximum value at the instant of maximum deformation to zero at the instant of just separation of bodies.

At the instant of maximum deformation the colliding bodies will have same velocity. Let the velocity of bodies at the instant of maximum deformation is $U_{D\,max}$

Applying Impulse-Momentum principle for first body

$$F_D dt = m_1 U_{D\,max} - m_1 u_1 \quad (1)$$

And $$F_R dt = m_1 v_1 - m_1 U_{D\,max} \quad (2)$$

Dividing Equation (2) by (1)

$$\frac{F_R dt}{F_D dt} = \frac{m_1 v_1 - m_1 U_{Dmax}}{m_1 U_{Dmax} - m_1 u_1}$$

i.e.

$$\frac{F_R dt}{F_D dt} = \frac{v_1 - U_{D\,max}}{U_{D\,max} - u_1}$$

(3)