Similarly for second body,

$$\frac{F_R dt}{F_D dt} = \frac{U_{D\,max} - v_2}{u_2 - U_{D\,max}} \tag{4}$$

From Equation (3) and (4)

$$\frac{F_R dt}{F_D dt} = \frac{v_1 - U_{D\,max}}{U_{D\,max} - u_1} = \frac{U_{D\,max} - v_2}{u_2 - U_{D\,max}}$$

$$= \frac{v_1 - U_{D\,max} + U_{D\,max} - v_2}{U_{D\,max} - u_1 + u_2 - U_{D\,max}}$$

$$= \frac{v_1 - v_2}{u_2 - u_1} = \frac{v_2 - v_1}{u_1 - u_2}$$

$$= \frac{Relative\ velocity\ of\ separation}{Relative\ velocity\ of\ approach}$$

Newton conducted the experiments and observed that when collision of two bodies takes place relative velocity of separation bears a constant ratio to the relative velocity of approach, the relative velocities being measured along the line of impact. This constant ratio is call as the co-efficient of restitution and is denoted by “e”. Hence

$$Coefficient\ of\ restitution = e = \frac{v_2 - v_1}{u_1 - u_2}$$

For perfectly elastic bodies the magnitude of relative velocity after impact will be same as that before impact and hence coefficient of restitution will be 1.

For perfectly plastic bodies the velocity of separation is zero as both bodies moving together after impact, so coefficient of restitution is zero.

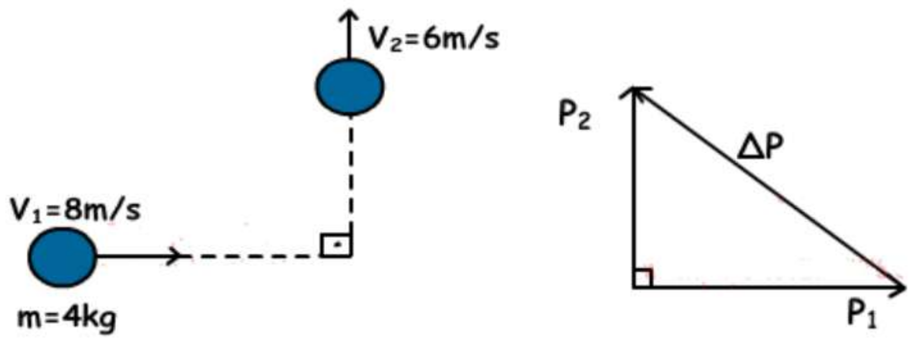
For semi-elastic collision where some part of energy conserved the value of coefficient of restitution in between 0 to 1.

So the coefficient of restitution always lies between 0 to1.The value of coefficient of restitution depends not only on the material property, but it also depends on shape and size of the body. Hence the coefficient of restitution is the property of two colliding bodies but not merely of material of colliding bodies.

Solved Examples:

Example 1: Ball having mass 4kg and velocity 8m/s travels to the east. Impulse given at point O, makes it change direction to north with velocity 6m/s. Find the given impulse and change in the momentum.
Solution:

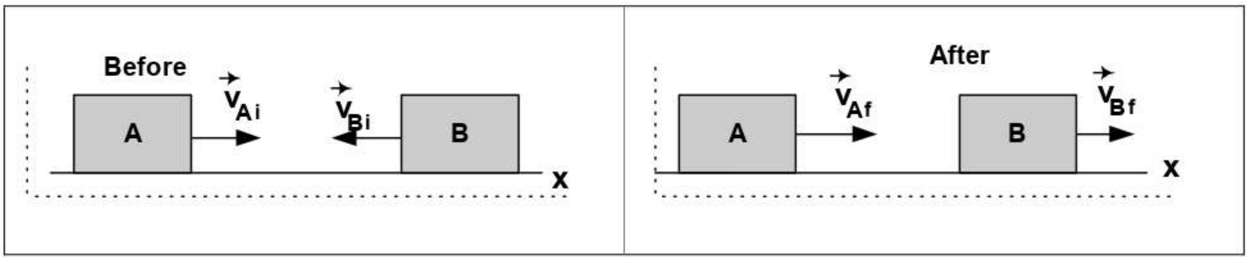
Initial and final momentum vectors of ball are shown in the figure below.



$P_1 = m.V_1 = 4kg.8m/s = 32kg.m/s$
$P_2 = m.V_2 = 4kg.6m/s = 24kg.m/s$
$\Delta P = P_2 + P_1$ (vector addition)
$\Delta P^2 = P_2^2 + P_1^2 = m^2(v_2^2 + v_1^2)$
$\Delta P^2 = 16.100$
$\Delta P = 40kg.m/s$
Impulse=change in momentum
$I = \Delta P = 40kg.m/s$

Example 2: Two blocks are travelling toward each other. The first has a speed of 10 cm/sec and the second a speed of 60 cm/sec. After the collision the second is observed to be travelling with a speed of 20 cm/sec in a direction opposite to its initial velocity. If the weight of the first block is twice that of the second, determine: (a) the velocity of the first block after collision; (b) whether the collision was elastic or inelastic.

Solution: We have a collision problem in 1-dimension. We draw both 'before' and 'after' pictures and select a coordinate system as shown.



Since the surface is frictionless, and since no work is performed by either mg or the normal, then the net force acting on the system is 0, and we have conservation of linear momentum:

Thus adding the x-components we have: $m_1 v_{1i} - m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

Since $m_1 = 2\, m_2$ we find: $2\, v_{1i} - v_{2i} = 2\, v_{1f} + v_{2f} \rightarrow (2)(10) - (60) = 2\, v_{1f} + 20$

Thus $2\ v_{1f} = -60$ and $v_{1f} = -30$ cm/sec ('-' means to left).

The initial KE is given by: $KE_I = (1/2)\ m_1\ (v_{1I})^2 + (1/2)\ m_2\ (v_{2I})^2$. This gives:

$= (1/2)(2\ m_2)(10)^2 + (1/2)\ m_2\ (60)^2 = (1/2)(200 + 3600)\ m_2 = 1900\ m_2$

The final KE is: $KE_f = (1/2)\ m_1\ (v_{1f})^2 + (1/2)\ m_2\ (v_{2f})^2$. This gives:

$= (1/2)(2\ m_2)(30)^2 + (1/2)\ m_2\ (20)^2 = (1/2)(1800 + 400)\ m_2 = 1100\ m_2$

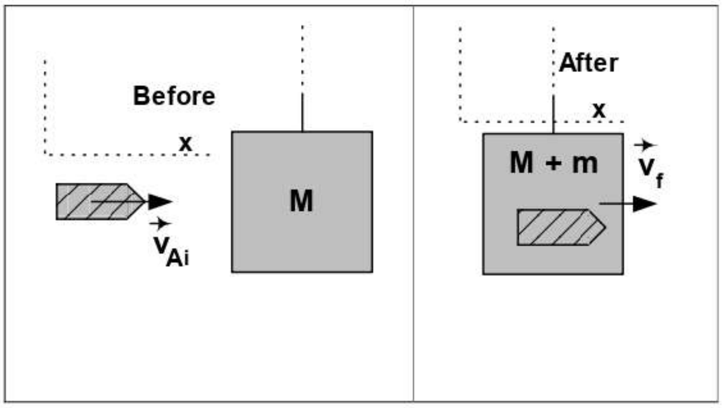
Since KE_f is not equal to KE_I, the collision is inelastic.

Example 3: A 0.005 kg bullet going 300 m/sec strikes and is imbedded in a 1.995 kg block which is the bob of a ballistic pendulum. Find the speed at which the block and bullet leave the equilibrium position, and the height which the center of gravity of the bullet-block system reaches above the initial position of the center of gravity.

Solution: A 'ballistic pendulum' is a device which can be used to measure the muzzle velocity of a gun. The bullet is fired horizontally into the block of wood and becomes imbedded in the block. The block is attached to a light rod and can swing like a pendulum. After the collision the 'bob' swings upward and the maximum height it reaches is determined. From this information plus the masses of the bullet and block, one can determine the velocity of the bullet. The critical point to note in this problem is that we have two distinct problems:

Problem 1: A collision problem. Apply Conservation of Linear Momentum and energy relation. Problem 2: A work-energy type problem. Apply conservation of total mechanical energy.

Part 1: We draw before&after pictures, label the velocities, and choose a CS. Conservation of total linear momentum is:



$m\mathbf{v}_{Ai} + M\ \mathbf{v}_{Bi} = (m + M)\ \mathbf{v}_f$.

In component form (for x-components) this gives:

$mv_{Aix} + M\ v_{Bix} = (m + M)\ v_{fx}$. Since $v_{Bi} = 0$, we have:

$v_f = (m\ v_{Ai}) / (m + M) = (.005)(300)/(2.00) = 0.75$ m/sec.

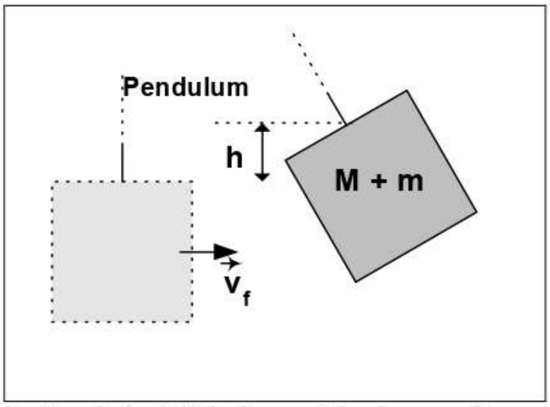
(Note: We were able to 'solve' the collision problem without an 'energy relation' since the collision was a perfectly inelastic collision. That is the two objects had the same final velocity. This condition is equivalent to an 'energy relation', since for such a collision the loss of KE is a maximum possible amount).

Part 2: In the work-energy part of the problem we note that the only force which performs work is gravity. Hence, we have only conservative forces present, and we have conservation of total mechanical energy.

We draw the figure indicating 'initial' and 'final' situations. We may choose the 0 level for gravitational potential energy anywhere we like. Hence, select $U_I = 0$.

Then: $KE_I + U_I = KE_f + U_f$.

$$(1/2)(m + M)\, v_f^2 = 0 + (m + M)g\, h$$



Here v_f is the 'initial' velocity in this part of the problem (0.75 m/sec) and (m+M) is the combined mass of bullet & block.

Thus: $h = v_f^2/2g = (.75)^2/(2)(9.8) = .0287$ m or 2.87 cm.

Note the reversal of this problem. If we know the masses and measure 'h', then from part 2 we can calculate 'v_f' (the initial velocity of bullet & block in the 2nd part of the problem). This is the same as v_f, the final velocity in the collision problem. Thus using this we can calculate v_{Ai} the 'muzzle' velocity of the bullet.

Example 4:A pile hammerweighing 20 kN drops from a height of 750mm on a pile of 10kN. The pile penetrates 100mm per blow. Assuming that the motion of the pile is resisted by a constant force, find the resistance to penetrate of the ground.

Solution: Initial velocity of hammer= u=0

Distance moved=h= 750mm=0.75m

Acceleration, g= 9.81m/sec^2

Velocity at the time of strike = $\sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.75}$ =3.836m/sec

Applying the principle of conservation of momentum to pie and hammer, we get velocity V of the pile and hammer immediately after the impact,

$$\frac{20}{9.81} \times 3.836 = \frac{20 + 10}{9.81} V$$

$$V = \frac{20}{30} \times 3.836 = 2.557 m/sec$$

Applying work energy equation to the motion of the hammer and pile, resistance R of the ground can be obtained,

$$(20 + 10 - R)s = \frac{20 + 10}{2g}(0 - V^2)$$

$$(30 - R)0.1 = \frac{20 + 10}{2 \times 9.81}(-2.557^2)$$

$$R = 130kN$$